



## Review article

# The influence of bending and shear stiffness and rotational inertia in vibrations of cables: An analytical approach

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## ABSTRACT

The main objective of this work is to analyze the influence of bending and shear stiffness, and rotational inertia in the natural frequencies of overhead transmission line conductors, and to compare the results with a vibrating string, where only geometrical stiffness is considered. Five formulations, based on Bernoulli's and Timoshenko's beam theories, taking into account the effects of geometrical, bending and shear stiffness and rotational inertia are considered here. They are also based on the assumption that the cable is inextensible. The equation of motion of the vibrating cable is developed analytically, assuming small strains. The Newton–Raphson method is employed for the solution of the nonlinear equations. A numerical example is presented and the results are compared with solutions obtained from the technical literature.

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## 1. Introduction

A very important application in the area of cable vibration that results in many studies and publications is the vibrating behavior of overhead transmission line conductors. A historical review of the string vibration problem was presented by Bassalo [1]. He emphasizes that during the years of 1732 and 1733 Daniel Bernoulli, John Bernoulli's son, studied small vibrations of a string of negligible weight, suspended in both ends and loaded with many equally spaced masses. Bassalo pointed out that Daniel had also worked with non-uniformly thick strings. According to Bassalo [1], D'Alembert in 1746 had studied the string vibration problem. He discussed the problem of a vibrating tensioned string, in which he arrived at a partial differential equation that describes the behavior

of a vibrating string. Irvine [2] pointed out that the first study about the shape of a suspended string subjected to its own weight is attributed to Galileo. The main contribution of this work was to present the similarity between this curve and a parabola. In 1691, a group of geographers and mathematicians obtained the analytical solution of this curve, which was called catenary. Ervik et al. [3] presented a review of the most important theoretical aspects of aeolic vibration, such as wind energy, conductor dumping, vibration level, tension, and turbulence effects on cables. They also presented studies about the stochastic properties of wind and its origin and behavior. Another very important aspect of this work was the derivation of analytical expressions for the calculation of conductor vibration levels. Abramovich and Elishakoff [4], following Timoshenko's suggestion, omitted the term representing the joint action of rotational inertia and shear deformation from the differential equations of motion. Abramovich and Elishakoff [5] commented that the classical Bernoulli–Euler theory of flexural behavior of an elastic beam has been known to be inadequate for vibration of higher modes and showed numerical examples

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