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Short communication

Refined Mindlin–Reissner theory of forced vibrations of shear deformable plates

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ABSTRACT

Two theories of shear deformable plate vibrations that account for the influence of the transverse normal stress component are derived. The first is a slightly modified Mindlin theory, including a shear correction factor, while in the second the use of the shear correction factor is replaced by stress components that exactly satisfy elasticity motion equations and plate face boundary conditions. A comparison with the exact 3D solution for a sinusoidally loaded rectangular plate and infinite strip is made. It is found that the refined theories significantly increase the accuracy of results compared to the original Mindlin theory.

1. Introduction

Because of its simple form, ease of prescribing boundary conditions, and reasonably accurate results, the Mindlin plate theory [1] is widely used for analyzing the vibrations of thick plates. Mindlin, who follows the works of Hencky [2] and Uflyand [3], and used Timoshenko's well known idea of the shear correction factor from the theory of bars, derived his equations straight from elasticity equations by using the following assumptions:

- the in-plane displacement components are linearly distributed across the plate's thickness;
- the deflection of the plate is constant across the plate's thickness;
- the transverse normal stress component is negligible.

How the Mindlin equations can be derived using variational calculus is presented, for one instance, in Liew's book [4]. To that we add that the bibliography of various extensions and generalizations of Mindlin's plate theory is quite extensive and its survey is beyond the scope of this paper. Interested readers should thus refer to the articles of Lewinski [5], Reddy [6], Goldenveizer et al. [7] and especcialy Liew et al. [8].

The aim of this paper is to extend a method of derivation of the basic equation of the equilibrium of plates present in one of the author's papers [9] to the case of plate dynamics. In that paper the Reissner and Mindlin equations of plate equilibrium were derived from elasticity equations without using variational calculus for the case of isotropic and transversally inextensible plates. In this paper only isotropic plates will be considered; more precisely, the equations will be derived without using the Mindlin assumption that the transverse normal stress component is negligible since this assumption, as was shown for the static case in [9] is unnecessary. As will be shown, the inclusion of this stress component yields additional terms in the plate's governing equations which are proportional to the gradient of the plate's face load. Besides that, an alternative derivation of dynamics plate equations is presented adding Reissner's assumption of linearly distributed in-plane stress components across the plate's thickness [10] to the form of displacements assumed by Mindlin. This extra assumption yields equations without the shear correction factor and contains additional inertial terms. Originally, Reissner considered only plate equilibrium; however, for convenience, this theory will here be called Reissner's theory.

The article is divided into four sections. In the first, the general equations of elasticity and plate theory are reviewed. The second section is devoted to the derivation of dynamics plate equations, discussion of their properties and some comportment with other theories. In the next section, a numerical example of forced vibration of a simply supported infinite strip is discussed in some detail. As will be shown, the refined equations, in the case of forced vibrations, lead to more accurate results than the original Mindlin equations. The article ends with a summary of conclusions.

2. General equations

Elasticity equations. Consider a plate of thickness h made from homogeneous and isotropic elastic material with the modulus of elasticity E and Poisson's ratio ν. For the description of a plate's elastic state a rectangular Cartesian coordinate system with coordinates (x, y, z) is used. The coordinate z is perpendicular to

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