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Upper and lower bounds for structural design of RC members with ductile response

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ABSTRACT

In the present paper, some of the complex phenomena characterizing the flexural behaviour of reinforced concrete beams, such as hyper-strength and snap-back and snap-through instabilities, are interpreted under a unified approach based on nonlinear fracture mechanics concepts. In particular, they are analysed by means of a numerical algorithm adopting the *cohesive crack model* for concrete in tension and the *overlapping crack model* for concrete in compression. According to the latter constitutive law, a fictitious interpenetration is assumed to describe the concrete damage, analogously to the fictitious crack opening used for tension.

Such an integrated cohesive-overlapping crack model is applied to assess the minimum reinforcement amount necessary to prevent unstable tensile crack propagation and to evaluate the rotational capacity of plastic hinges. The main novelty is given by the capability to predict the size-scale effects evidenced by several experimental programmes available in the literature. According to the numerical results obtained, new practical design formulae and diagrams are proposed, as well as, upper and lower bounds to the reinforcement amount, the material properties and the structural dimensions are defined in order to avoid brittle failures.

1. Introduction

The global response of reinforced concrete (RC) beams in bending during the loading process is characterized by complex phenomena due to mechanical nonlinearities, as evidenced in the qualitative moment vs. rotation diagram shown in Fig. 1. In more details, we refer to concrete fracturing, which produces a hyper-strength in the increasing branch, steel yielding, which is at the origin of a ductile behaviour, and concrete crushing, which determines a decrease in the load carrying capacity and, consequently, a limit to the ultimate rotation.

Although two failure modes are usually observed in the flexural behaviour, i.e., yielding of the steel reinforcement and crushing of the compressed concrete, more complex phenomena can affect the load vs. displacement relationships: snap-through instability, defined as a loss of stability in the controlled load condition, and snap-back instability, representing a loss of stability in the controlled displacement condition. Such phenomena are very general, and usually encountered in structural problems characterized by either geometrical or mechanical nonlinearities. As an example, they may appear in the buckling response of elastic structures, as evidenced by von Kármán and Tsien [1] for thin cylindrical shells under axial compression, and by Carlson et al. [2] and Kaplan [3] for complete spherical shells and spherical caps subjected to external pressure. On the other hand, snap-back instabilities can be easily encountered when materials exhibiting strain softening behaviours are considered. This is, for instance, the case of plain concrete slabs in tension and in bending, whose overall responses are highly influenced by the softening behaviour of the process zone ahead of the real crack tip. The detailed analytical and numerical investigations carried out by Carpinteri [4,5] by means of the *cohesive crack model* put into evidence a transition from softening to snap-back instability either by increasing the specimen dimensions and/or the material strength, or by decreasing the material fracture energy. The virtual post-peak catastrophic branch, characterized by a positive slope in the load vs. displacement plane, can be captured only if the loading process is controlled by the crack mouth opening displacement. In this context, the application of the cohesive crack model to three-point-bending tests on plain concrete beams has permitted to describe the size effects on the nominal flexural tensile strength, σ_n, function of the maximum load in the hypotheses of linear strain distribution along the cross-section and linear-elastic behaviour for concrete in tension and compression. The numerical results, as a function of the beam height, are compared to the empirical prescriptions provided by the Model Code 90 [6] and the Eurocode 2 [7] in Fig. 2. The ratio σ_n/σ_u, where σ_u is the limit stress that the material can locally sustain and, therefore, is a material property, tends to unity only for very large heights, h,

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