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Recent developments in optimal reinforcement of RC beam and column sections

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ABSTRACT

Recent advances in approaches to the design of reinforced concrete sections have culminated in a theorem of optimal (minimum) sectional reinforcement. This theorem is articulated on the basis of patterns observed in the optimal reinforcement of rectangular sections, obtained with a new approach for the analysis and design of reinforcement. Using the hypotheses for ultimate strength design sanctioned by ACI 318-05 (2008), the minimum total reinforcement area required to provide adequate resistance to axial load and moment is shown to occur for particular constraints on longitudinal reinforcement area or distributions of strain. These constraints are identified along with the solutions for minimum total reinforcement area. Optimal reinforcement may be selected from among the potential solutions identified by the theorem. An example illustrates the application of the theorem to the design of a reinforced concrete cross-section. Implications for teaching and practice of reinforced concrete design are discussed.

1. Introduction

The strength design of reinforced concrete sections subjected to combinations of axial load and moment applied about a principal axis of the cross-section is well understood based on approaches that were developed many years ago. Other approaches should be considered if flexural behavior or confinement influence wants to be studied [1–3]. The strength design understanding, however, has been challenged by a new design approach, presented by Hernández-Montes et al. [4]. The recent approach allows conventional solutions to be seen as special cases of a more general family of solutions. The general family of solutions, which can be generated for arbitrary combinations of imposed axial load and moment, include so-called optimal (or minimum) reinforcement solutions, which, in general, require non-symmetric distributions of reinforcement over the cross-section. An understanding of the characteristics of these optimal solutions has led to the development of the theorem of optimal section reinforcement presented herein.

The longstanding conventional approaches to treat the strength design of sectional reinforcement under uniaxial bending are classified in Fig. 1. There is a general consensus that if the axial force is zero, a solution may be obtained that includes only bottom (tension) reinforcement. If the depth of the beam is limited, a doubly reinforced section may be required. In either case, a non-symmetric distribution of longitudinal reinforcement is customary for the design of sections with zero axial force.

In cases with non-zero axial forces, one may either consider symmetric reinforcement, as is done routinely for columns, or non-symmetric reinforcement. Symmetric reinforcement is usually considered. One approximate approach (for $A_s = A'_s$) distinguishes between large and small eccentricities as suggested by Whitney (see the textbook by Nawy [5]). An exact treatment of the symmetric reinforcement case is realized with normalized P–M interaction charts (e.g. [6] and SP-17 from ACI Committee 340 [7]. These design-oriented charts are based on N–M interaction diagrams, which have been widely adopted since their initial presentation by Whitney and Cohen [8]. The design of axially loaded sections having non-symmetric reinforcement is rarely discussed—Walther and Miehlbardt [9], for example, provide only a particular solution in this instance.

Experience with beam design suggests that an asymmetric pattern of reinforcement would be more economical for small axial loads for cases in which the applied moment (or eccentricity) acts in one direction only. As [4] have shown, the family of solutions for combinations of top and bottom reinforcement required to confer adequate strength to a cross-section constitutes an infinite set of solutions that includes the symmetric reinforcement solution obtained using conventional P–M interaction diagrams. The family of solutions may be displayed graphically on a "reinforcement sizing diagram" (RSD). Unlike a P–M interaction diagram, an RSD displays the family of reinforcement solutions applicable to a specified combination of bending moment and axial load.

RSDs were used in a recent investigation by Aschheim et al. [10] to characterize the optimal (minimum) reinforcement solutions

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