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Stability of columns with semi-rigid connections including shear effects using Engesser, Haringx and Euler approaches

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ABSTRACT

A complete column classification and the corresponding stability equations that can be used to evaluate the elastic critical axial loads of prismatic columns with sidesway uninhibited, partially inhibited, and totally inhibited including the simultaneous effects of bending and shear deformations and semi-rigid connections are presented and derived using three different approaches. The first two approaches are those by Engesser and Haringx that include the shear component of the applied axial force proportional to the total slope (dy/dx) and to the angle of rotation of the cross-section (ψ) along the member, respectively. The third approach is a simplified formulation based on the classical Euler theory that includes the effects of shear deformations but neglects the shear component of the applied axial force along the member. Comparisons among the critical loads obtained using these three approaches and those using the classical Euler solutions for classical column cases are presented. Examples that demonstrate the effectiveness and accuracy of the proposed stability equations and the importance of shear deformations and shear component of the applied axial force are presented in detail. These effects must be taken into account particularly in structures made of columns with relatively low effective shear areas A_s (like laced columns, columns with batten plates or with perforated cover plates, and columns with open webs) or with low shear stiffness GA_s of the same order of magnitude as $\pi^2 EI/h^2$ (like short columns made of laminated composites with low shear modulus G when compared to their elastic modulus E). These effects become even more significant when the external supports are not perfectly clamped. The effects of semi-rigid connections and those due to shear forces and deformations are condensed into the proposed characteristic equations.

1. Introduction

The combined effects of shear forces and deformations along with bending on the critical axial load of prismatic columns were first studied by F. Engesser in 1891 and later by F. Nussbaum in 1907. Timoshenko and Gere [1] (pp. 132–144) present these effects on the static lateral buckling of prismatic and built-up columns (laced, with batten plates, and with perforated cover plates) along with the historical contributions of Engesser, Nussbaum, Prandatl, Olsson, Haringx, and others. Additional details are given by Bazant and Celodin [2] (pp. 30, 102 and 734).

In framed structures shear deflections, the shear component of the applied axial force as each member deflects, as well as the lateral deflections (P–δ effects) along each element and the relative drift between the element ends (P–Δ effects) all cause nonlinear behavior with additional bending moments, rotations and displacements. These nonlinear geometrical effects not only alter the lateral stiffness of each element and that of the structure as a whole, but also the buckling capacity of each element and that of the entire structure. For instance, the P–δ effects may lead to individual members buckling, while the P–Δ effects to overall structural instability or story buckling. The combined effects of axial force and shear deflections can also cause the phenomenon of buckling under tension forces which was studied experimentally and reported by Kelly [3] in multilayer elastomeric bearings and later discussed by Aristizabal-Ochoa [3,4].

The elastic stability of prismatic columns with sidesway uninhibited, partially inhibited, and totally inhibited including the effects of flexural deformations were presented in a classical manner by Aristizabal-Ochoa [5–10]. It was demonstrated that the stability equation of the partially braced column was the "missing link" to a full understanding of the elastic stability of columns and frames and the "key" to solve the limitations of current methods. Because of their non-paradoxical results in asymmetrical frames and in frames with leaning columns [6], the validity of the proposed method was extended to 2-D and 3-D multi-column systems with semi-rigid connections [7–10]. More recently, the stability and second-order analysis of 3D multi-column systems including the simultaneous effects of bending and

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