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Model for post-yield tension stiffening and rebar rupture in concrete members

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ABSTRACT

A tension stiffening model is presented which enables the calculation of average tensile stresses in concrete, after yielding of reinforcement, in reinforced concrete elements subjected to uniaxial tension, shear or flexure. To determine the average tensile stress–strain relationship for concrete, a crack analysis approach is employed taking into account the bond mechanism between concrete and deformed reinforcing bars, and numerical analyses are conducted to determine the tensile behavior of reinforced concrete members including post-yield response. Analytical parametric studies are conducted to determine the influence of various parameters including concrete compressive strength and reinforcement yield strength, ultimate strength, hardening stress, and hardening strain. Analysis results obtained from the proposed model, when compared to experimental results for uniaxial members, indicate good agreement for structural behavior after yielding of reinforcement. The proposed model makes it possible to accurately calculate reinforcement stresses at crack locations and, thus, average strain conditions which result in rupture of reinforcement. This leads to more realistic predictions of the uniaxial, flexural, and shear ductility of reinforced concrete members.

1. Introduction

In a reinforced concrete member that has suffered cracking, the contribution of average post-cracking tensile stresses in the concrete must be appropriately considered in order to accurately predict the structural behavior of the member. After the tensile stress capacity of the concrete is exceeded and the member cracks, the concrete tensile stress at a crack drops to zero as the tension softening response is exhausted, but the concrete between cracks still attracts tensile stresses due to the effects of bond between the concrete and the reinforcing steel; this mechanism is known as tension stiffening. One approach to considering tension stiffening effects in the tensile behavior of concrete members is to formulate an average tensile stress–strain relationship for the concrete; this approach has been successfully used in finite element analysis [1] and sectional analysis [2] of concrete structures.

A variety of tension stiffening models have been developed for evaluating the average tensile stress of concrete [3–8]. Most have focused on the tensile behavior before yielding of reinforcement to predict the response illustrated with line *A*–*B* in Fig. 1 where *A* and *B* represent initial cracking of the concrete and yielding of the reinforcement at a crack, respectively. [In Fig. 1, the difference between the tensile stress in a concrete-embedded bar and that in a bare bar is defined as the tension stiffening effect.] In addition, researchers recently investigated the effect of concrete shrinkage [9] and the bond–slip relationship [10] on the tension stiffening behavior, and presented an analytical procedure and a modified bond model, respectively.

In previous models [4–8], the equilibrium condition at a crack is typically checked using the yield strength of steel reinforcement as the limit capacity. After local yielding of the reinforcement at a crack, the average tensile stress of concrete due to tension stiffening is reduced because the total strength capacity across the crack is assumed to not exceed the yield strength of the reinforcement (line *B*–*C* in Fig. 1 where *C* denotes the point where the average tensile strain is equal to the yield strain of the reinforcement). (Unless this equilibrium check is performed, the capacity of the concrete member may be unconservatively evaluated due to an overestimation of concrete tensile stress.) Since local strains of the reinforcement in the vicinity of the crack abruptly increase without significant increase in the tension, points *B* and *C* in Fig. 1 do not coincide. With the crack equilibrium check imposed, after the average strain of the reinforcement reaches the yield strain, the average tensile stress of concrete becomes zero. Hence, once the average stress of the reinforcement has reached yield, the contribution of concrete on tensile behavior is typically ignored; the computed tensile behavior of a reinforced concrete member after yielding of reinforcement becomes the same as that of bare steel bars. Consequently, the customary equilibrium check at a crack results in the member's calculated

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