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Sequentially linear modelling of local snap-back in extremely brittle structures

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ABSTRACT

In the present paper a sequentially linear approach is proposed for the finite element analysis of extremely brittle structures. A peculiar definition of the saw-tooth approximation of the softening branch is presented, which allows for a straightforward discretization of the softening branch in the case of snap-back on a constitutive level. In this way the well-known limit of the smeared crack approach is overcome, and extremely brittle structures, i.e. with a very low ratio between the fracture energy and the finite element size, can be modeled without the need of an extreme mesh refinement. The model is applied to a novel 'reinforced glass beam' concept by the Faculty of Architecture at the Delft University of Technology. The sequentially linear analysis results in terms of load–deflection curves and crack patterns, obtained with different relatively coarse meshes, are in good agreement with the experimental results.

1. Introduction

The sequentially linear approach in modelling strain softening materials and structures has recently been demonstrated to be effectiveespecially when the ratio between the elastic energy stored and the energy that can be dissipated by fracture is very large, i.e. in the case of real scale structures [1]. The model approach replaces the downward stress–strain curve by a saw-tooth curve, either saw-tooth tension-softening for unreinforced material or saw-tooth tension stiffening for reinforced material. A linear analysis is performed, the most critical integration point is traced, the stiffness and strength of that integration point are reduced according to the saw-tooth curve, and the process is repeated. The sequence of scaled critical steps provides the global load–displacement response.

In the present paper we consider some experimental results obtained from bending of a reinforced three-layer glass beam. The mechanical response of the beam is extremely brittle, despite the presence of reinforcement at the beam intrados, since the glass fracture energy is very small and the span of the beam quite large. Therefore, this structure can be considered representative of a class of extremely brittle structures. Due to the very limited ductility of the structure, the phenomenon of snap-back takes place several times during the loading procedure [2].

From a numerical point of view, one possibility could be to model the fracture occurrence in glass by means of discrete interface elements, where the cohesive law is given in terms of crack aperture. Unfortunately, this approach is not well suited in the case of reinforced structures, since many cracks arise during the tests and the position and direction of cracks cannot be easily determined in advance. On the other hand, smeared cracking can be adopted, which provides automatic detection of the crack pattern. The dilemma is the role of the negative slope of the softening branch. Incorporating the negative slope in the tangent stiffness of a Newton–Raphson scheme to solve the systems of equations, might turn out to be crucial for finding localizations of cracks along with a structural snapback behavior. At the same time the use of the negative slope in the tangent stiffness might make the analysis less stable, if not unstable. When the fracture energy divided by the crack bandwidth is very small, snapback behavior even shows up on a constitutive level. A rigorous numerical treatment of the softening becomes impossible. From a practical point of view, apart from adapting the material parameters, the only way out with extremely brittle structures is to perform the analysis with an extremely refined mesh, which is often unaffordable due to the required computational effort.

In this paper it is shown that adopting the sequentially linear approach can effectively solve the problem. In fact, a mesh-independent saw-tooth discretization of the softening branch can be used even in the case of snapback behavior on a constitutive level, allowing for affordable coarser mesh discretizations. The formulation of the method, a validation with a benchmark problem and an illustration by a comparison with experimental results are provided and commented.

2. The overall "event-by-event" procedure

The locally brittle snap-type response of many brittle and quasi-brittle structures inspired the idea to capture these brittle

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