



# Buckling of cylindrical shells with stepwise variable wall thickness under uniform external pressure

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## ABSTRACT

Cylindrical shells of stepwise variable wall thickness are widely used for cylindrical containment structures, such as vertical-axis tanks and silos. The thickness is changed because the stress resultants are much larger at lower levels. The increase of internal pressure and axial compression in the shell is addressed by increasing the wall thickness. Each shell is built up from a number of individual strakes of constant thickness. The thickness of the wall increases progressively from top to bottom.

Whilst the buckling behaviour of a uniform thickness cylinder under external pressure is well defined, that of a stepped wall cylinder is difficult to determine. In the European standard EN 1993-1-6 (2007) and Recommendations ECCS EDR5 (2008), stepped wall cylinders under circumferential compression are transformed, first into a three-stage cylinder and thence into an equivalent uniform thickness cylinder. This two-stage process leads to a complicated calculation that depends on a chart that requires interpolation and is not easy to use, where the mechanics is somewhat hidden, which cannot be programmed into a spreadsheet leading to difficulties in the practical design of silos and tanks.

This paper introduces a new “weighted smeared wall method”, which is proposed as a simpler method to deal with stepped-wall cylinders of short or medium length with any thickness variation. Buckling predictions are made for a wide range of geometries of silos and tanks (unanchored and anchored) using the new hand calculation method and compared both with accurate predictions from finite element calculations using ABAQUS and with the current Eurocode rules. The comparison shows that the weighted smeared wall method provides a close approximation to the external buckling strength of stepped wall cylinders for a wide range of short and medium-length shells, is easily programmed into a spreadsheet and is informative to the designer.

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## 1. Introduction

Cylindrical shells of stepwise variable wall thickness are widely used for cylindrical containment structures, such as vertical-axis tanks and silos. The increase of internal pressure and axial compression in the shell is addressed by increasing the wall thickness [1]. Each shell is built up from a number of individual strakes of constant thickness. The thickness of the wall increases progressively from top to bottom. Fig. 1 shows a typical example of a tank structure taken from ECCS EDR5 [2].

The thickness of the wall of these cylindrical vessels is generally very thin and the strength is often controlled by elastic buckling failure. The buckling behaviour of a cylindrical shell with uniform wall thickness under uniform external pressure has been extensively studied by many researchers [3–7]. However, the mechanics of buckling in cylinders of stepwise variable wall thickness has not

been captured well in current design procedures and has drawn little attention to date. This may be partly because the formulation of general rules for all possible patterns of thickness variation is difficult.

In the European Standard for Shells [8] and European Recommendations on Shell Buckling [2], stepped wall cylinders under circumferential compression are transformed, first into a three-stage cylinder and thence into an equivalent uniform thickness cylinder based on the research of Resinger and Greiner [9–11]. This two-stage process leads to a complicated calculation that depends on a chart that requires interpolation and is not easy to use, where the mechanics is somewhat hidden, and which cannot be programmed into a spreadsheet so that difficulties arise in the practical design of silos and tanks.

This paper introduces a new “weighted smeared wall method”, which is proposed as a simpler method to deal with stepped-wall cylinders of short or medium-length with any thickness variation. The method is developed from an idea proposed by Trahair et al. [12] for stepped wall cylinders under external pressure. Buckling predictions are made for a wide range of geometries for silos and both unanchored and anchored tanks using the new

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