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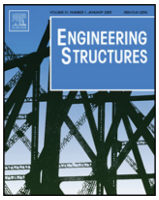

On the background and biresonant components of the random response of single degree-of-freedom systems under non-Gaussian random loading

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ABSTRACT

The variance of the response of a single degree-of-freedom system subjected to a low frequency excitation is usually decomposed into its background and resonant contributions. In this paper we aim at the formulation of such a decomposition for the third statistical moment, which should in principle sidestep the heavy double integration of the bispectrum of the response. In large finite element models, the estimation of the bispectrum of the loading for a given frequency (ω_1, ω_2), which is a necessary stage towards estimation of the response, is the most expensive computational task. We therefore formulate the problem with the underlying constraint that the number of estimations of the bispectrum of the force be kept as small as possible. Invoking the perturbation theory in the context of the computation of integrals, we propose to decompose the third moment into background and biresonant components with expressions that are not trivially adapted from the decomposition of the variance. Thanks to the proposed method, the double integration of the bispectrum is avoided and represents accurately the response of lightly to moderately damped structures under a low-frequency loading. The formal expression for the biresonant component still requires an integration that should preferably be avoided. In the second part of the paper, we then investigate the practical implementation of the proposed formula and study the possible application of local, global or hybrid numerical approximations of that remaining integral, so as to further increase the computational efficiency. Finally two numerical experiments illustrate the prospect of the proposed method: the third statistical moment of the response is accurately computed with less than 20 estimates of the bispectrum of the loading, whereas an advanced numerical procedure for the double integration would require a mesh of probably more than (a) thousand(s) points.

1. Introduction

Many engineering applications consist in determining the *steady state* statistics of the output of a *linear* time invariant system subjected to a *Gaussian* stationary random input. This well posed problem has been studied meticulously since 1950's and formal solutions of this problem may nowadays be found in the literature (e.g. [1]). A typical application -commonly called buffeting analysis- in wind engineering deals with the study of gusty wind flows around structures [2,3]. In this field, a series of papers published by Davenport in the early 1960's [4,5] is usually recognized as the incipient engagement of the beneficial pooling of applied mathematics, statistics, meteorology and structural dynamics. In his work, Davenport formulated the necessary assumptions that allowed his concept of buffeting analysis fall within the typical framework of that class of problems.

Even if the framework of this class of problems appears to be restrictive since it requires (i) stationarity, (ii) system linearity and (iii) Gaussianity, the majority of applications still hinge on this concept. The reason is that the application of this theory is rather simple and that the suppression of any of the three limitations brings the problem to a much higher level of complexity, which is typically not affordable to practitioners.

Formal solutions to the enlarged class of problems (considering non-stationarity, nonlinearity and/or non-Gaussianity) have however existed since the 1950's [6]. Ito proposed to tackle this more general problem by means of stochastic differential equations which allow to cope with any or several of the above limitations. Nevertheless, stochastic differential equations are doubtlessly not suitable for everyday applications. The transfer of knowledge from pure mathematics to the engineering community took a couple of decades as it required these solutions to be presented in a simpler and applicable form (e.g. [1]). Actually, several analysis methods emerged concurrently with formulations simpler than Ito's stochastic differential equations, but encompassing only one of the three refinements at a time. For instance and setting again our sights on wind engineering applications, nonstationary random wind loadings are now used to model downbursts [7,8] and their influence on structures; random nonlinear structural behavior is also sometimes considered [9–12]; and non-Gaussianity of

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