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# Equivalent representations of beams with periodically variable cross-sections

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ABSTRACT

This paper examines representations of a beam with periodically variable cross-sections. First, the rotational equivalence of a beam with two periodical steps to a uniform beam is considered when a constant bending moment is applied along the lengths of the two beams. The representation is then extended to a beam with several different steps and then to a beam with a variable cross-section. Variable bending moments are also taken into account in the equivalent representation. Equivalent representations based upon the same maximum deflection and fundamental natural frequency are derived. If a beam consists of several periodically variable units and the bending moment in each unit is considered to be constant, the various equivalent criteria can be unified. Detailed finite element analysis (FEA) shows that the plane-deformation assumptions no longer hold in the transition regions between adjacent steps. The transition area is defined using a straight line with an angle determined by curve-fitting and the results are provided in a tabular form considering all related parameters. A comparison of the maximum displacement and fundamental natural frequency shows good agreement for a number of beams with periodically variable cross-sections and their equivalent uniform beams. The study demonstrates that the equivalent representation of a beam with periodical variable cross-sections is accurate and simple to apply to engineering practice.



## 1. Introduction

A stepped beam is a beam with abrupt changes of cross-section and/or material properties. A beam with a variable cross-section is an extension of a stepped beam as it also includes beams with continuous changes of cross-section. Such beams are frequently used in aeronautical engineering (e.g. rotor shafts), mechanical engineering (e.g. robot arms and crane booms) and civil engineering (e.g. tall buildings).

A practical example is composite floors that consist of a trapezoidal steel sheet and lightweight concrete. This leads to difficulties in finite element modelling because of the two different materials and variable cross-sections. In one study [1] 2700 solid elements and 2700 shell elements were used for modelling a 4.5 m × 3.0 m composite panel. On this basis modelling the complete 45 m × 21 m multi-panel floor would require 180,000 solid and shell elements, i.e. over 360,000 nodes and 2160,000 degrees of freedom. It is obvious that this is not an economical or feasible way to model such a floor. If the profiled composite floor can be reduced to an equivalent flat floor, then modelling the floor would be significantly simplified and become a realistic option for engineering practice. One feature of the profiled floor is that the cross-section varies periodically in one of the two main directions, which can be thought to be a stepped beam in that direction. If the properties of an equivalent uniform beam can be determined, the composite floor can then be simplified to an equivalent orthotropic flat floor. This forms the background for this study of equivalent representations of a beam with periodically variable cross-sections.

In the last few decades, efforts have been made to obtain an analytical solution for the dynamic behaviour of stepped beams. Jang and Bert [2,3] developed a method of undetermined coefficients to study the free vibration of a stepped beam. The deflection function of a segment in a stepped beam was expressed as a general solution with four undetermined coefficients from the motion equation. The total $4n$ coefficients in $n$ segments were solved using (a) the $2(n-1)$ continuity equations of deflection and slope at the junctions; (b) the $2(n-1)$ compatibility equations of moment and shear force at the junctions, and (c) the four boundary conditions at the ends of the stepped beam. The natural frequency of the fundamental mode of a stepped beam with two segments for four boundary conditions (pinned–pinned, clamped–clamped, clamped–free and clamped–pinned) was given in Ref. [2] and for the higher modes in Ref. [3].

Maurizi and Belles [4,5] extended the method of undetermined coefficients to a stepped beam with elastic transverse and rotation support at one end [4] and both ends [5]. De Rosa [6] added an elastic vertical support and a torsional support to the junction of the two segments in Maurizi's model. The continuity of

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