



# Efficient modeling of imperfections for buckling analysis of composite cylindrical shells

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## ABSTRACT

Unstiffened composite cylindrical shells show a large scatter in the load levels that the structure can withstand before buckling occurs. Such scatter is greatly influenced by the unavoidable imperfections of the structure, introduced during the fabrication phase. It is thus of key importance to be able to accurately model such imperfections in a numerical computation, in order to recreate and predict the scatter shown in experimental buckling tests. The imperfections can be analyzed by means of random fields, inferring their statistical properties from available measurements. In this manuscript, evolutionary spectra are used to derive random fields of surface and material imperfections of cylindrical shells. A procedure based on a moving window averaging technique is proposed in order to accurately capture the variation of material properties due to imperfect thickness and laminate manufacturing. Finally, Monte Carlo simulations of compression and torsional buckling of cylinders are carried out to show the combined effect of surface and thickness imperfections in the scatter of the buckling limit load.

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## 1. Introduction

Buckling is the most dangerous failure mode for unstiffened cylindrical shells under axial compression, since the buckled cylinders can no longer carry the load that was acting on the structure before the buckling occurrence. Thus, it is crucial to design such cylinders according to the buckling condition. However, a very large scatter in the limit load is observed in experiments; the reason for this scatter is a combined effect of geometric non-linearities and imperfection sensitivity of the cylindrical shells.

Koiter pioneered in his doctoral thesis [1] the study of stability of cylindrical shell and the effect of imperfections on buckling. The particular type of imperfection used in his work, indicated now as “classical”, was modeled by means of deviations of the surface of the shell from the nominal radius. The functional evolution of such deviations was sinusoidal in axial and circumferential directions, where the amplitude and wave number of the sinusoids were variables [2–4].

Realistic imperfections, however, show shapes that greatly differ from the one considered in the classical analysis [5,6]. Nowadays, highly sophisticated numerical solvers allow one to compute the buckling load of shells with no need of assumptions

on the shape of imperfections. Data-bank of imperfections [5] are available and can be used to capture the statistical features of the geometric imperfections, described by means of random field. Additionally, non-classical imperfections may have an effect on the buckling load such as non-perfect boundaries, misalignment in the loading, fluctuation in the material properties, etc. Consideration of both classical and non-classical imperfections allows one to accurately recreate through Monte-Carlo simulations the scatter observed in the experimental results by cylindrical shells constructed in isotropic materials [7–10].

Cylindrical shells made of composite materials introduce new challenges in the study of the buckling phenomenon. As a matter of fact, the anisotropic nature and the complex manufacturing of such materials introduce variability in material properties and thickness imperfections greater than the one found in isotropic shells. As an example, composites can show uneven resin distribution, pores or an inter-laminar gap with the consequence of significant variations in the stiffness properties of the laminate. A computational framework that efficiently tackles these additional imperfections it is thus necessary.

An efficient strategy to model the involved imperfections, as well as the different effect of these imperfections in the buckling of shells are analyzed in this manuscript. Section 2 introduces the available methods to estimate such spectra from the available realizations of the random field, used in the model of surface imperfections. A moving-window averaging technique, used to infer the material properties fluctuation in the composite shells

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