A new estimator based on likelihood function for drift time of change in Poisson rate parameter

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***Abstract*— Although a control chart can signal an out-of-control state in a process, but it does not always indicate when the process change has begun. Identifying the real time of the change in the process, called the change point, is very important for eliminating the source(s) of the change and assists process engineers in identifying the responsible special cause and ultimately in improving the process.**

In this paper, we first introduce an estimator for a change point with linear trend in the Poisson process, based on the likelihood function using a slope parameter. Then we apply Monte Carlo simulation to evaluate the accuracy and the precision performance of the proposed change point estimator. Finally we compare, the proposed estimator with the MLE of the Poisson process change point derived under linear trend disturbance on the basis of cumulative sum (CUSUM) and Shewhart C control charts. The results show that the proposed procedure outperforms the MLE designed for drift time with regard to variance and is more effective in detecting drift time when the magnitude of change is relatively large.

Keywords- Quality control; Statistical process control; Change point; Poisson process; c chart; CUSUM charts

I. INTRODUCTION

Statistical process control (SPC) has played an important role in industry for many years. After examining the process, SPC uses statistical tools to find the sources of variation in the process parameter(s). Control charts are used to monitor for changes in a process by distinguishing between special causes and common causes of variation. When a control chart signals an out-of-control alarm, process engineers initiate a search for the assignable cause disturbing the process. By signaling, control charts do not provide specific information regarding the cause nor when the process changed; rather, they only suggest that a change has occurred. Cumulative sum (CUSUM) control charts suggested by Page were among the primary tools for detecting change in the process [7]. Though they were not as simple to operate as Shewhart control charts but they have been shown to be more efficient in detecting small shifts in the process mean. Exponentially weighted moving average (EWMA) control charts also offered a procedure for estimating the process change point [5]. Despite their ability in detecting small shifts, they were not quite effective tools for finding large shifts in the process mean. Knowing the exact point of change in a process would help to search and identify special causes, resulting in time saving to find the causes. Therefore, it is useful to identify the difference between the change point and the time when an out of control signal is generated by control charts.

Poisson count processes are often used to model the number of occurrences over some interval unit. The interval unit can be time, distance, area, volume or some similar unit. Often in an industrial quality control setting, the Poisson distribution is used to model the number of defects or nonconformities per unit of product. That is, the probability that a randomly selected unit of product contains x nonconformities is given by

$$P(X = x) = \frac{\lambda^x}{x!} \cdot e^{-\lambda}$$

Where $x \geq 0$, and $\lambda \geq 0$ denotes the mean rate of nonconformities. The on-line monitoring of λ is typically accomplished through the use of a SPC chart. For monitoring Poisson rates, the c, CUSUM and EWMA control charting procedures are most commonly used.

In the current research first, a model for a linear trend change in the rate parameter of a Poisson process is introduced and a new estimator for detect the drift time of change is proposed. It uses the change likelihood function for a linear trend disturbance. Next, the proposed estimator is compared with the MLE of the drift change in Poisson process following the signals from c and CUSUM control charts.