International Journal of Non-Linear Mechanics 46 (2011) 1090–1091

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Note on similarity solutions for viscous flow over an impermeable and non-linearly (quadratic) stretching sheet

Neil A. Kelson

High Performance Computing and Research Support Group, Division of Technology, Information, and Learning Support, Queensland University of Technology, Australia

ARTICLE INFO

Article history:
Received 14 February 2008
Accepted 7 April 2011
Available online 5 May 2011

Keywords:
Similarity solutions
Non-linearly stretching sheet

ABSTRACT

Similarity solutions for flow over an impermeable, non-linearly (quadratic) stretching sheet were studied recently by Raptis and Perdikis (Int. J. Non-linear Mech. 41 (2006) 527–529) using a stream function of the form $\psi = \alpha x f(\eta) + \beta x^2 g(\eta)$. A fundamental error in their problem formulation is pointed out. On correction, it is shown that similarity solutions do not exist for this choice of ψ.

1. Introduction

Mathematical and numerical studies of so-called continuous surface flow due to a stretching sheet have received a large amount of attention in the literature. The early work of Crane [1] found simple closed form solutions for the case of linear stretching. Later, the experimental data of Vleggaar [2] indicated that in certain practical applications the assumption of linear stretching may have limited validity. This has motivated a number of studies to consider a quadratic stretching function at the surface of the sheet, starting with the early work of Takhar et al. [3] and Kumaran and Ramanaiah [4] to the more recent studies of Kumar Khan and Sanjayanand [5], Siddheshwar et al. [6] and Raptis and Perdikis [7].

It is the recent work of Raptis and Perdikis that is of interest here. In contrast to the other studies cited above, they consider an *impermeable* rather than permeable sheet, and a stream function (ψ) of the form $\psi = \alpha x f(\eta) + \beta x^2 g(\eta)$ is used in a seemingly successful attempt to find similarity solutions. Their work is briefly reviewed below for the purpose of showing that the theoretical development has omitted, erroneously, an equation from the analysis, which completely invalidates their findings.

2. Mathematical formulation and solution

Raptis and Perdikis considered steady boundary layer flow over an *impermeable* and non-linearly (quadratic) stretching sheet. Additionally, they included a source term in the momentum equation to account for magnetic field effects acting on an electrically conducting fluid. Their mathematical formulation was

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{1}$$

with associated boundary conditions

$$u(y=0) = ax + cx^2, \quad v(y=0) = 0 \text{ and } \lim_{y\to\infty} u = 0 \tag{2}$$

Here u and v are the respective flow velocity components in the x and y directions, ν is the kinematic viscosity, ρ is the fluid density, σ is the electrical conductivity, B_0 is the magnetic field strength and a, c are positive non-zero constants.

In addition to the above, Raptis and Perdikis also included a mass transfer equation and associated boundary conditions in their problem description. However, the mass transfer equation will not be discussed here, as its inclusion or otherwise will be seen to be incidental to the observations made below.

In an attempt to find similarity solutions, Raptis and Perdikis used the transformations

$$\eta = \sqrt{\frac{a}{\nu}}y, \quad \psi = \sqrt{a\nu}f(\eta) + \frac{cx^2}{\sqrt{a/\nu}}g(n) \tag{3}$$

where $u = \partial\psi/\partial y$ and $v = -(\partial\psi/\partial x)$. On substitution into Eqs. (1) and (2) above, the following coupled ordinary differential equations are obtained:

$$f''' + ff'' - (f')^2 - Nf' = 0 \tag{4}$$

$$g''' + fg'' - 3f'g' + 2f''g - Ng' = 0 \tag{5}$$

$$gg'' - (g')^2 = 0 \tag{6}$$

E-mail address: n.kelson@qut.edu.au

0020-7462/$ - see front matter Crown Copyright © 2011 Published by Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijnonlinmec.2011.04.025