International Journal of Non-Linear Mechanics 46 (2011) 286–290

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Implicit constitutive relations in thermoelasticity

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ARTICLE INFO

Article history:
Received 13 February 2010
Received in revised form
11 August 2010
Accepted 17 September 2010

Keywords:
Heat flux vector
Implicit constitutive relation
Conduction
Fourier's law
Thermoelasticity

ABSTRACT

In this paper, after a brief discussion on the implicit constitutive relations used in thermoelasticity, based on Fox's [1] work, we derive a general implicit relation for the heat flux vector. In the section following that we use the methodology suggested by Rajagopal [2,69] whereby we derive a class of implicit constitutive relations for $\mathbf{q}$ and we show that by selecting appropriate functions appearing in the formulation, we can obtain as special cases the Fourier heat conduction and the Maxwell–Cattaneo–Fox model. We do not discuss the implications of the second law of thermodynamics.

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1. Introduction

The concept of boundary layers for the deformation field in solid materials, in the sense of strains being larger adjacent to a solid boundary, has been studied in details recently (see [4]). These regions are often known as strain localization regions. Interestingly, non-linear elastic materials whose shear modulus depends on the deformation in an appropriate manner can be a cause for the presence of boundary layers. At the same time, it is well known that many polymeric and rubber-like solids shear stiffen due to increase in temperature. Thus, when the shear modulus is a function of temperature, one can discuss the possibility of boundary layers at elevated temperatures (see [5]). An example of this kind in fact was given in the book by Treloar [6], *Physics of Rubber Elasticity*, where the shear modulus μ in a new-Hookean solid is defined through $\mu = kN\theta$, where k is Boltzmann's constant, N is the number of network junctions, and θ is the absolute temperature (see also [7]). Rajagopal [4] indicates that when the material moduli, especially the shear modulus, depends on the temperature and the stretch, there are competing or reinforcing effects whereby the boundary layer structure can either be annihilated or strengthened. In all the cases presented by Rajagopal [4] the heat flux vector in the energy equation was assumed to be given by Fourier's (conduction) equation (see also [8,9]).

The heat flux vector is commonly modeled by Fourier's law of heat conduction [10] and for complex materials such as non-linear fluids, porous media, or granular materials, the coefficient of thermal conductivity is generalized by assuming that it would depend on a host of material and kinematical parameters such as temperature, shear rate, porosity or concentration, etc. In classical linearized thermoelasticity (see [11]), the heat flux vector is modeled as

$$\mathbf{q} = \mathbf{q}(\mathbf{e}, \theta, \mathbf{g}) \tag{1}$$

where
$\mathbf{e} = 1/2(\mathbf{F}\mathbf{F}^T - \mathbf{1}); \mathbf{g} = \nabla\theta; \mathbf{F} = \mathbf{1} + \nabla\mathbf{u}$ and $\mathbf{u}$ is the displacement vector. This relationship for an isotropic thermoelastic body reduces to the classical Fourier's assumption, where

$$\mathbf{q} = -k \operatorname{grad} \theta \tag{2}$$

where k is generally assumed to be constant. In recent years, due to manufacturing of new materials such as fiber-reinforced composites, microfabrication technologies, nanoscale thermal transport especially in semiconductor industry, micro-time heat transfer processes such as short-pulse laser applications and high-speed electronics (see [12–15]), there has been a growing need for more accurate representation and understanding of conductive and radiative heat transfer processes. In fossil fuel applications, for example, the self-heating of coal stockpiles has a long history of posing significant problems to coal producers because it lowers the quality of coal and may result in hazardous thermal runaway. Precise prediction of the self-heating process is, therefore, necessary in order to identify and evaluate control measures and strategies for safe coal mining, storage, and transportation.

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0020-7462/$ - see front matter Published by Elsevier Ltd.
doi:10.1016/j.ijnonlinmec.2010.09.009