International Journal of Non-Linear Mechanics 46 (2011) 1191–1194

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Slip flow due to a stretching cylinder

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ARTICLE INFO

Article history:
Received 28 July 2010
Accepted 20 May 2011
Available online 1 June 2011

Keywords:
Slip flow
Stretching cylinder
Similarity

ABSTRACT

The slip flow due to a stretching cylinder is studied. A similarity transform reduces the Navier–Stokes equations to a set of non-linear ordinary differential equations. Asymptotic solutions for large Reynolds number and small slip show the problem can be related to the existing two-dimensional stretching cases. Due to algebraic decay, the equations are further transformed through a compressed variable, and then integrated numerically. It is found that slip greatly reduces the magnitudes of the velocities and the shear stress.

1. Introduction

The viscous flow due to a stretching boundary occurs in the extrusion of metal, plastic and food products. The stretching causes the entrainment of the adjacent fluid, which in turn affects the resistance and the solidification of the extruded material. If the extrusion velocity is linear with respect to distance [1], which usually occurs with material subjected to constant strain, similarity solutions of the fluid problem may exist [2]. The two-dimensional stretching of a flat sheet was solved by Crane [3] and the axisymmetric radial stretching of a surface by Wang [4]. Brady and Acrivos [5] considered the flow inside a stretching channel or tube, while Wang [6] studied the flow outside a stretching cylinder. The above sources, and their many extensions, applied the no-slip boundary condition between the fluid and the extrusate.

In certain cases, the no-slip condition does not hold and should be replaced by a partial slip condition. These occur when the fluid is a rarefied gas [7], or when it is a particulate such as blood, foam, emulsion or suspension [8]. Slip also occurs on hydrophobic surfaces, especially in micro- and nano-fluidics [9].

Two-dimensional stretching surface with partial slip was studied by Andersson [10] and Wang [11], and the axisymmetric case by Ariel [12]. It was found that slip affects the velocities and the fluid resistances considerably. The purpose of the present paper is to investigate the axisymmetric similarity solution due to a stretching of a cylinder with a partial slip boundary. The results are relevant to the extrusion of filaments which are micron size and/or in a rarefied environment. It is also an exact similarity solution of the Navier–Stokes equations.

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doi:10.1016/j.ijnonlinmec.2011.05.014

2. Formulation

Fig. 1 shows a cylinder of radius a being stretched longitudinally with a surface velocity of $Ws=2bz$ where b is a constant and z is the axial direction. In addition, there is a longitudinal free stream velocity of W at infinity. We use a transformation similar to that of Wang [6]

$$u=-ab\frac{f(\eta)}{\sqrt{\eta}},\quad w=2bzf'(\eta)+Wg(\eta),\quad \eta=\left(\frac{r}{a}\right)^2 \tag{1}$$

Here (u, w) are velocities in the (r, z) directions, respectively, and continuity is satisfied. Since there is no longitudinal pressure gradient, using Eq. (1), the Navier–Stokes equations reduce to the non-linear ordinary differential equations

$$\eta f'''(\eta)+f''=R[(f')^2-ff''] \tag{2}$$

$$\eta g''(\eta)+g'=R(gf'-fg') \tag{3}$$

where $R=ba^2/2\nu$ (ν being the kinematic viscosity) is the Reynolds number.

On the cylinder surface, the partial slip condition [13] states that the slip velocity u_s is proportional to the local shear stress τ:

$$u_s=N\tau \tag{4}$$

where N is the slip coefficient. Eqs. (1) and (4) then yield

$$f'(1)-1=\lambda f''(1) \tag{5}$$

$$g(1)=\lambda g'(1) \tag{6}$$

Here $\lambda=2N\rho\nu/a$ (ρ being the fluid density) is the normalized slip factor. Zero radial velocity on the surface requires

$$f(1)=0 \tag{7}$$