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A practical method for the evaluation of eigenfunctions from compound matrix variables in finite elastic bifurcation problems

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ABSTRACT

We show how the compound matrix method can be extended to give eigenfunctions as well as generalised eigenvalues to bifurcation problems in non-linear elasticity. When the incremental problem is formulated in terms of displacements only there are significant difficulties that arise from the non-trivial boundary conditions. In order to avoid these problems we adopt a Stroh formulation of the incremental problem. This then produces trivial boundary conditions for the compound matrix eigenvalue problem and more importantly known initial conditions for the compound matrix eigenfunction problem. This results in a straightforward and robust calculation for the eigenfunctions.

1. Introduction

In an earlier attempt at this problem [1,2] it was shown how the compound matrix method could be extended to give eigenfunctions as well as eigenvalues for fourth and sixth order problems in solid mechanics and in particular bifurcation problems in non-linear elasticity. The method for fourth order problems involved solving a second order system with one known initial condition (found simply by a normalisation condition) and one unknown condition. A simple shooting method to satisfy a target condition worked very well. For the sixth order problem the derivation of the eigenfunctions required the solution of a third order system with one normalised initial condition and two unknown initial conditions. We again required a shooting method to achieve a given target condition at the other end of the range. While this method could be made to work and give reasonable solutions there was a considerable effort required in finding initial approximations to the unknown initial conditions, so much so that the method could not really be recommended as a practical proposition.

It has been found that the trivial boundary conditions naturally associated with similar problems in fluid mechanics lead to a much simpler calculation for the eigenfunctions, see [3–5]. With this in mind, we re-examine the solid mechanics bifurcation problem but we now focus attention on the boundary conditions for the original eigenvalue problem. We show that adopting a Stroh formulation of the bifurcation problem, see [6], and references therein, for example, leads to trivial initial conditions and a trivial target condition for the standard compound matrix eigenvalue problem. This is of no real consequence for the eigenvalue problem, however, it does lead to a different approach to the eigenfunction problem. To determine the three components of the incremental displacements we now have to solve a sixth order initial value problem with known initial conditions rather than the third order system with shooting for two unknowns that we had in [2]. In this case we also solve for the (possibly unwanted) incremental stresses. One novel feature of the present method is that we incorporate the coefficients from the original equations whereas other approaches to the eigenfunction problem exclusively use the compound matrix variables. For some problems of interest a single incremental displacement is all that is required. In this case we show how we may isolate a single displacement and solve a third order system for the normalised displacement (and two stresses). For comparison the determinantal method (see [2]) solves a third order system three times and then solves a standard eigenvalue and eigenvector problem for a three by three matrix to obtain all three incremental displacements.

In Section 2 we demonstrate the Stroh formulation of a typical bifurcation problem from non-linear elasticity and how the basic compound matrix problem is then formulated. Following this we describe the compound matrix eigenvalue method and apply it to a particular problem. Throughout we make comparisons with the previous attempt at this problem [2]. We employ cylindrical coordinates in anticipation of the specific example to be used but the underlying method is clear.

2. Incremental problem and the compound matrix method

In the absence of body forces the incremental equilibrium equations can be written

$$\operatorname{div} \dot{\mathbf{s}}_0 = \mathbf{0}, \tag{1}$$

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