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Exact solutions for the incompressible viscous magnetohydrodynamic fluid of a rotating disk flow

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A R T I C L E I N F O

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A B S T R A C T

The main interest of the present investigation is to generate exact solutions to the steady Navier–Stokes equations for the incompressible Newtonian viscous electrically conducting fluid flow motion due to a disk rotating with a constant angular speed. For an external uniform magnetic field applied perpendicular to the plane of the disk, the governing equations allow an exact solution to develop taking into account of the rotational non-axisymmetric stationary conducting flow.

Making use of the analytic solution, exact formulas for the angular velocity components as well as for the wall shear stresses are extracted. It is proved analytically that for the specific flow the properly defined thicknesses decay as the magnetic field strength increases in magnitude. Interaction of the resolved flow field with the surrounding temperature is further analyzed via the energy equation. The temperature field is shown to accord with the dissipation and the Joule heating. According to Fourier's heat law, a constant heat transfer from the disk to the fluid occurs, though decreases for small magnetic fields because of the dominance of Joule heating, it eventually increases for growing magnetic field parameters.

1. Introduction

The cases when an exact solution for the Navier–Stokes equations can be obtained, are of particular interest in investigations to describe fluid motion of the viscous fluid flows. However, since the Navier–Stokes equations are non-linear in character, there is no known general method to solve the equations in full, nor the superposition principle for non-linear partial differential equations does work. Exact solutions, on the other hand, are very important for many reasons. They provide a standard for checking the accuracies of many approximate methods such as numerical or empirical. Although, nowadays, computer techniques make the complete integration of the Navier–Stokes equations feasible, the accuracy of the results can be established only by a comparison with an exact solution.

The Navier–Stokes equations were intensively studied in the literature commencing with the paper [1], see also the review by Constantin in [2]. There are well-known exact unidirectional or parallel shear flows, a few sample cases contain the steady Poiseuille and Couette flow. In addition to this, there are exact cylindrically symmetric solutions with closed plane streamlines, see for instance [3,4]. Further exact solutions that we already know possess certain feature of the fluid motion, such as rectilinear motion, motions of the duct flows, axisymmetric flows and stagnation flows on plate with slip, etc., see for instance, the works of [5,6].

Von Karman's swirling viscous flow [7] is a well-documented classical problem in fluid mechanics, which has several technical and industrial applications. The original problem raised by von Karman, which is the most studied by researchers in the literature, is the viscous flow motion induced by an infinite rotating disk where the fluid far from the disk is at rest. Then the problem is generalized to include the case where the fluid itself is rotating as a solid body far above the disk. Another generalization is to consider the viscous flow between two infinite coaxial rotating disks. All these problems and also stability issues are attacked, theoretically, numerically and experimentally, by many researchers amongst many others, such as [8–19]. Additionally, heat transfer problem over a rotating disk was also studied, see for instance [20–22].

While the flow of a fluid due to the rotation of parallel plates or disks is assumed usually to have axially symmetric solutions, [23] in his study of the flow between parallel planes rotating with the same constant angular velocities about a common axis exhibited a one parameter family of solutions only one of which is axisymmetric. Later on, [24] proved the existence of a one parameter family of solutions when the planes are rotating with constant but different angular velocities about a common axis or about non-coincident axes. Similar to the parallel disks, it was shown in [25] that the flow of a fluid due to the rotation of a single disk also admits solutions that do not possess axial

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