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Mathematical modelling of rate-independent pseudoelastic SMA material

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ABSTRACT

The paper presents a procedure for the formulation of constitutive equations for rate-independent pseudoelastic SMA material models. The procedure applies a rheological scheme representing mechanical properties of the material. An additive decomposition of strains into two parts is proposed. The first part describes strains of a perfectly elastic body while the second part may be represented by a combination of a rigid perfectly elastic body and a rigid perfectly plastic body. It is demonstrated that the key problem of formulation of constitutive relationships is to derive the 1st order differential equation with respect to the tensor describing the second part of the strain field. This equation may be obtained in explicit form starting from the variational inequalities defining non-elastic parts of rheological model. The uniqueness of the obtained differential equation has been proved. A numerical implementation of the constitutive relationships of SMA material was done through the user subroutine module VUMAT within the FE commercial code ABAQUS/Explicit. As an example we analyzed the problem of vibration of a simple 3D structure made of SMA.

1. Introduction

A strong non-linear response of the shape memory alloys (SMAs) is associated with the two special effects: shape memory and pseudoelasticity. The shape memory effect allows material to recover its shape upon heating. The pseudoelasticity phenomenon makes the material capable of experiencing large inelastic strains recoverable upon unloading. In the literature there is a comprehensive description of SMA behaviour associated with the stress- or temperature-induced transformations of their crystalline structure [1,2].

The response of the structures made of SMA is characterized by the energy dissipation capabilities as well as by the shape recovery along the loading path. As a result, SMA materials are now widely used in engineering. The main non-medical applications of SMA, recently explored are: protection of civil constructions against earthquake damages [3–5] and control of space structures such as antennas and satellites [6].

The mathematical modelling of 1D and 3D systems based on SMA has received a great deal of attention in the literature. The model presented by Lubliner and Auricchio [7] uses the notion of theory of plasticity. An additional internal variable, so called phase fraction, was used there. The problem of numerical implementation of 3D thermomechanical SMA material was considered by Auricchio and Petrini [8]. A robust integration algorithm was adopted there for finite element method (FEM) implicit applications. Bernardini and Vestroni [9] investigated a non-linear dynamic behaviour of a pseudoelastic oscillator. Another 1D dynamical system made of the SMA bar with an additional mass was analyzed by Feng and Li [10]. Auricchioet al. [11] presented a uniaxial rate-dependent viscous model suited for seismic applications.

The objective of our paper is to formulate the constitutive equations of pseudoelastic SMA material model suited for reproducing its hysteretic behaviour. The model is purely phenomenological without considering any phase transformation properties. The problem will be defined within the notion of classical small strain theory of plasticity [12]. The rate-dependency phenomenon will not be considered. The main advantage of the method being proposed herein is that the differential equations defining the behaviour of the material are of explicit type. Thus, the existence of the solution as well as its uniqueness may be proved. On the other hand, it allows straightforward implementations in the FEM commercial codes.

Another novel finding of our approach is that the 3D constitutive equations of SMA are formulated using a rheological scheme representing its mechanical properties. We demonstrate the procedure comparing the SMA scheme with the classical elastoplastic scheme (a combination of a spring element and a

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