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Non-Newtonian pseudoplastic fluids: Analytical results and exact solutions

M. Guedda [a,*], R. Kersner [b]

[a] LAMFA, CNRS UMR 6140, UPJV, Amiens, France
[b] University of Pécs, PMMK, Pécs, Hungary

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ABSTRACT

A one layer model of laminar non-Newtonian fluids (Ostwald–de Waele model) past a semi-infinite flat plate is revisited. The stretching and the suction/injection velocities are assumed to be proportional to $x^{1/(1-2n)}$ and x^{-1}, respectively, where n is the power-law index which is taken in the interval $(0,\frac{1}{2})$. It is shown that the boundary-layer equations display both similarity and pseudosimilarity reductions according to a parameter γ, which can be identified as suction/injection velocity. Interestingly, it is found that there is a unique similarity solution, which is given in a closed form, if and only if $\gamma=0$ (impermeable surface). For $\gamma \neq 0$ (permeable surface) we obtain a unique pseudosimilarity solution for any $0 \neq \gamma \geq -((n+1)/3n(1-2n))^{n/(n+1)}$. Moreover, we explicitly show that any pseudosimilarity solution exhibits similarity behavior and it is, in fact, similarity solution to a modified boundary-layer problem for an impermeable surface. In addition, the exact similarity solution of the original boundary-layer problem is used, via suitable transverse translations, to construct new explicit solutions describing boundary-layer flows induced by permeable surfaces.

1. Introduction and main results

Many problems in boundary-layer theory lead to particular exact solutions which are characterized by similarity velocity profiles and agree with experimental observations and numerical simulations. It is the purpose of this work to investigate possible conditions for similarity and pseudosimilarity solutions to a class of the boundary-layer flows of laminar non-Newtonian fluids. The range of non-Newtonian fluid behavior exhibited by industrial liquids is very large and the mathematical formulation is, in general, complex. A broad description of the behavior in both steady and unsteady flow situations, together with mathematical models, can be found for example, to mention a few, in Astin et al. [1], Astarita et al. [2], Barnes [3], Bird [4], Tanner [5], Schowalter [6], Rajagopal et al. [7] and Rajagopal [8,9]. The most frequently used model in non-Newtonian fluid mechanics is the Ostwald–de Waele model, or the non-Newtonian power-law fluid, for which the shear stress τ is related to the strain rate u_y via the expression [10–21]:

$$\tau = \nu |u_y|^{n-1} u_y, \tag{1}$$

where the subscript y denotes the partial derivative with respect to y,ν is a positive constant and $n>0$ is the power-law index. The case $n<1$ is referred to as pseudoplastic or shear-thinning fluids, and the case $n>1$ is known as dilatant or shear-thickening fluids. The Newtonian fluid is a special case where the power-law index n is one.

To begin with, we give a brief description of the problem. Consider a steady boundary-layer flow due to a moving plane surface in a quiescent fluid. The fluid can be injected or sucked. For the first approximation, the model is described by the Prandtl or the boundary-layer equations for non-Newtonian power-law fluids [10–21]:

$$\begin{cases} u_x + v_y = 0, \\ u u_x + v u_y = \nu(|u_y|^{n-1} u_y)_y, \end{cases} \tag{2}$$

with the boundary conditions:

$$\begin{cases} u(x,0) = u_w(x), \quad v(x,0) = v_w(x), \\ \qquad u(x,\infty) = \lim_{y\to\infty} u(x,y) = 0. \end{cases} \tag{3}$$

The Cartesian coordinates (x,y) are such that the $x \geq 0$ coordinate is along the plate and the $y \geq 0$ coordinate is normal to it with $y=0$ is the plate. Subscripts x and y denote partial derivatives with respect to those variables and u and v are the velocity components along the x- and y-axes, respectively. The stretching and suction/injection velocities are assumed to be of the form:

$$u_w(x) = U_0\left(\frac{x}{l}\right)^m, \quad v_w(x) = V_0\left(\frac{x}{l}\right)^p, \; p = \frac{m(2n-1)-n}{n+1}, \tag{4}$$

where $n \neq \frac{1}{2}$ and $m=(1/(1-2n))(p=-1)$. U_0 and V_0 are constants and the number l (the characteristic length) is the distance x where the stretching velocity equals U_0. It is assumed that the positive x-direction is that of the main stream, so that $U_0>0$. The

* Corresponding author.
E-mail address: guedda@u-picardie.fr (M. Guedda).

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