



# Instability of Bingham fluids in Taylor–Dean flow between two concentric cylinders at arbitrary gap spacings

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## ABSTRACT

Stability of Bingham fluids is investigated numerically in azimuthal pressure-driven flow between two infinitely long concentric cylinders. An infinitesimal perturbation is introduced to the basic flow and its time evolution is monitored using normal mode linear stability analysis. An eigenvalue problem is obtained which is solved numerically using pseudo-spectral collocation method. Numerical results are obtained for two different cases: (i) the inner cylinder is rotating at constant velocity while the outer cylinder is fixed (i.e., the Taylor–Dean flow) and (ii) both cylinders are fixed (i.e., the Dean flow). The results show that the yield stress always has a stabilizing effect on the Taylor–Dean flow. But, for the Dean flow the effect of the yield stress is predicted to be stabilizing or destabilizing depending on the magnitude of the Bingham number and also the gap size.

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## 1. Introduction

In 1923, Taylor [1] discovered that circular Couette flow may lose its stability depending on the gap size and the sense of rotation of the two concentric cylinders with respect to each other – the so-called Taylor–Couette instability. Similarly, in 1928 Dean [2] showed that pressure-driven flow between two fixed concentric cylinders may undergo centrifugal instability under certain conditions – the so-called Dean instability. Due to their engineering applications, both types of instabilities have been the subject of many studies in the past [3–5]. While the main application of Taylor–Couette instability appears to be in relation to Couette and/or Searl viscometers, application of Dean instability appears to be much broader encompassing heat and mass transfer enhancement, chaotic mixing, microfluidics, and micro-filtration, among others [6–13]. Instability of the combined flow (the so-called Taylor–Dean flow) has also attracted much attention in the past due to its application in hydrodynamic instability theories [14,15]. For understandable reasons, most studies carried out in the past in relation to centrifugal instability have treated the fluid as a Newtonian fluid. This is irrespective of the fact that most industrial fluids are non-Newtonian. There are also works published in the literature in which the non-Newtonian behavior of the fluid has been taken into account. Among different non-Newtonian fluids, viscoelastic fluids have been studied more extensively in the past [16–25] perhaps due to their importance

in polymer industry. Another fluid of interest, which has been addressed in previous studies, is the yield-stress fluids. The interest in such fluids arises from the fact that, yield stress appears to be a rule rather than an exception in industrial fluids. In fact, it is long established that complex fluids such as food-stuffs, pharmaceuticals, cosmetics, paints, and drilling muds exhibit yield stress. A well-known model to represent yield-stress fluids is the Bingham model [26]. Instability of Bingham fluids has been addressed in the past in certain geometries including Poiseuille flow [27–31], and circular Couette flow [32–35]. More recently, Fellouah [36], Soleimani [37], and Soleimani and Sadeghy [38] have investigated Dean instability of Bingham fluids in curved ducts and concentric cylinders. The major finding of these studies is that the yield stress has a stabilizing effect on the flow.

To the best of our knowledge, instability of Bingham fluids between two infinitely long concentric cylinders with one of the cylinders rotating at constant velocity while the other cylinder is being kept fixed has not previously been addressed using normal mode analysis. In the present work, we are going to investigate hydrodynamic instability of Bingham fluids in pressure-driven flow between two fixed concentric cylinders of infinite length when the inner cylinder is rotating and the outer cylinder is fixed. Our approach is a classical one in which we superimpose an infinitesimal perturbation to the basic flow and see how its amplitude evolve in time—the so-called temporal instability analysis. An eigenvalue problem will be obtained this way in which terms non-linear in the perturbation quantities will be dropped for ease of analysis—the so-called linear instability analysis. The analysis proceeds based on the assumptions that the perturbations are axisymmetric and can be represented by

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