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Interactions between vortices and flexible walls

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ABSTRACT

We give two fundamental solutions for the motion of a point vortex near a flexible wall, up to first order in wall deflection, using computational methods. For a point vortex near an infinite horizontal wall, the deformation of the wall intensifies the flow at the wall near the vortex, and increases the speed of the vortex. Near a circular wall there is a strong mutual amplification of the deflection of the wall and the pressure force induced by the deflection, as the point vortex approaches the wall. The total force on the wall diverges as the inverse cube of the distance to the point vortex, and the induced speed of the point vortex diverges as the inverse fourth power of distance to the wall.

1. Introduction

The interaction of vortices with solid walls is a classical problem in hydrodynamics [1], with recent applications in problems of biological and technological interest. Doligalksi et al. [2] review studies of flows past aircraft and submarines, where vorticity shed from upstream structures (i.e. airframes and helicopter blades) collides with downstream surfaces, causing boundary layer separation and leading to dramatic changes in unsteady forces. Rockwell's review [3] gives many additional examples including vortices impinging on small bodies, leading edges, and oscillating bodies, in the presence of three-dimensional effects and non-uniform background flows. Many important vortex–body interactions occur in biological flows, both internal (heart flows [4]) and external (insect flight [5–7] and fish swimming and schooling [8,9]). Many of these biological structures undergo large deformations under forces induced by vortices. The goal of the present work is to obtain two of the most basic solutions for vortices interacting with deformable walls. These solutions can be regarded as a starting point for a wider class of interactions of vortices with flexible walls which incorporate boundary layer interactions, different vorticity distributions such as dipoles, and more complex geometries. Interactions of vorticity with passive flexible flag-like structures have also been studied experimentally [10], theoretically, and computationally [11–14].

We consider a two-dimensional flow consisting of a single point vortex translating along a flexible wall which is either an infinite line or a circle in the undeformed state. We solve the problem in an asymptotic limit of small deformations. For large deformations, boundary layer separation is likely, which would inject additional vorticity into the outer flow. The leading order wall deformation can be determined from the unperturbed flow, which can be solved using classical methods such as the method of images [1]. To understand how the wall deformation alters the motion of the vortex and the force on the wall, we use a more general formulation in terms of bound vortex sheets. We find that the wall deformation increases the speed of the vortex as it travels along the wall, and increases the force on the wall near the vortex. The total force on the wall is either unchanged (for the infinite wall) or increased (for the circular boundary) by the wall's deformation. As the distance between the point vortex and the circular wall is decreased, the first-order correction to the flow grows rapidly, due to a mutual amplification of the body's deformation and the fluid forces on the wall.

2. Point vortex near a flexible wall

We first give the equations for the motion of a single point vortex immersed in an inviscid fluid above an infinite flexible wall. The position ζ of the wall in the complex plane is given by its vertical deflection h from the horizontal axis, $\zeta(x,t)=x+ih(x,t)$ (see Fig. 1a). In what follows, we assume $h(x,t)$ is small compared to the length scale of the problem, which is the distance of the point vortex from the wall, d. Then we retain terms up to linear order in h and $\partial_x h$ and drop terms which are $O(h^2,\partial_x h^2)$. Some of the details of this expansion are given in [15,16]. The method given here can also be used for more general $\zeta(x,t)$.

The boundary condition that the flow does not penetrate the wall can be satisfied by placing a vortex sheet at the wall, which induces a normal velocity along the wall. The vortex sheet also induces a tangential velocity along the wall, which is equal to the strength of the vortex sheet [17]. The flow everywhere above the wall is then the superposition of the potential flows induced by

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