International Journal of Non-Linear Mechanics 46 (2011) 577–585

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Treadmilling swimmers near a no-slip wall at low Reynolds number

Darren Crowdy

Department of Mathematics, Imperial College London, 180 Queen's Gate, London SW7 2AZ, United Kingdom

A R T I C L E I N F O

Available online 28 December 2010

Keywords:
Low Reynolds number
Swimmers
Confined geometries
Treadmilling

A B S T R A C T

The motion of a circular treadmilling low Reynolds number swimmer near a no-slip wall is studied analytically. First, the exact solution of Jeffrey and Onishi [Q. J. Mech. Appl. Math., 34 (1981)] for a translating and rotating solid cylinder near a no-slip wall is rederived using a novel conformal mapping approach that differs from the original derivation which employed bipolar coordinates. Then it is shown that this solution can be combined with the reciprocal theorem, and the calculus of residues, to produce an explicit non-linear dynamical system for the treadmilling swimmer's velocity and angular velocity. The resulting non-linear dynamical system governing the swimmer motion is used to corroborate the qualitative results obtained by an approximate model of the same swimmer recently presented in Crowdy and Or [Phys. Rev. E., 81 (2010)].

1. Introduction

The study of organisms swimming at low Reynolds number near a no-slip wall has received significant attention in recent years [4,19]. Much work has focussed on quantifying the swimming speeds and energetics when swimmers travel near solid boundaries [26,15,16,11]. More recent investigations have addressed the detailed dynamics of confined swimmers and the subtle interplay between the time evolution of a swimmer's orientation and its position. One well-known qualitative feature of their dynamics is that low Reynolds number organisms tend to be attracted to no-slip surfaces [27,34,11,6,35,13,2]. Rothschild [27] carried out an early experimental investigation of this phenomenon; more recently, Berke et al. [2] repeated Rothschild's experiment and proposed that the observed wall attraction is a hydrodynamic mechanism in which wall interactions cause a swimmer to change its orientation in such a way that it is eventually attracted to its image in the wall. Other related work is due to Smith et al. [30] who performed numerical calculations involving simple models of sperm cells to examine their dynamics near a no-slip wall. Shum et al. [29] carried out a numerical study of how the particular geometrical characteristics of swimmers can affect their dynamics near walls.

As well as attracting organisms, no-slip surfaces can also cause them to exhibit more interesting dynamics. The phenomenon of "swimming in circles" is one example and, recently, a hydrodynamical explanation for that behaviour has been put forward by Lauga et al. [20]. Another example is the recent experimental observation [10] that, as they draw close to solid surfaces, *Volvox* algae interact with those surfaces, and each other, to engage in various kinds of periodic hydrodynamic bound states. Drescher et al. [10] refer to these bound states as "dancing" and they include periodic motions of sufficient spatio-temporal complexity that they are dubbed the "waltz" and the "minuet". *Volvox* is a class of organisms operating at low Reynolds numbers which move, in part, by virtue of the imposition of a tangential surface velocity caused by motion of cilia on a nearly spherical body.

Other studies have revealed additional features of a swimmer's behaviour near a straight wall. Or and Murray [22] conducted numerical experiments to understand wall-bounded swimmer motion from a control and dynamical systems perspective. They considered model swimmers comprising small networks of spheres attached by rigid rods and actuated by rotation of the spheres about axes perpendicular to the plane of motion of the configurations. Or and Murray [22] computed the dynamics using a model by Swan and Brady [33] who computed approximations to the mobility matrices for multiple interacting spherical particles in the presence of a flat plane wall. Or and Murray [22] identify several interesting features of a swimmer's dynamics including steady states in which it travels at uniform speed at a constant distance from the wall. The generic swimmer motion along the wall is, however, along other non-linear periodic orbits having a more complicated spatio-temporal structure. Their numerical observations have since been corroborated by laboratory experiments involving small robotic swimmers in a tank of viscous fluid [37].

In order to rationalize the observations in [22,37] a simple two-dimensional model of a low Reynolds-number swimmer near a wall has been given by Crowdy and Or [9]. They performed an approximate analysis of a circular treadmilling swimmer, of the kind first considered by Blake [3], actuated by imposing a tangential velocity field on its boundary; this is an approximation, within an "envelope model", of the macroscopic effect of moving cilia on the swimmer's body. When in isolation, such a swimmer is shown

E-mail address: d.crowdy@imperial.ac.uk

0020-7462/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijnonlinmec.2010.12.010