International Journal of Non-Linear Mechanics 46 (2011) 247–252

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Towards a new type of energy trap: Classical analog of quantum Landau-Zener tunneling

Leonid I. Manevitch [a], Yury A. Kosevich [a], Mercedes Mane [b], Grigori Sigalov [b], Lawrence A. Bergman [b], Alexander F. Vakakis [b,*]

[a] Semenov Institute of Chemical Physics RAS, 4 Kosygin St., 119991 Moscow, Russia
[b] College of Engineering, University of Illinois at Urbana-Champaign, 104 South Wright St., Urbana, IL 61801, USA

A R T I C L E I N F O

Article history:
Received 31 December 2009
Received in revised form
4 August 2010
Accepted 23 August 2010

Keywords:
Targeted energy transfer
Energy trap
Coupled parametric pendulums
Landau-Zener tunneling

A B S T R A C T

We present a novel type of an energy trap providing targeted energy transfer in a system of weakly coupled pendulums. Our approach is based on the analogy we revealed between the behavior of two weakly coupled classical parametric pendulums (in linear approximation) and the nonadiabatic Landau-Zener tunneling in a two-state quantum system. This analogy leads us to the prediction of an efficient irreversible transfer of vibration energy from one subsystem to another, when the eigenfrequency of at least one of them changes in time, so that the coupled subsystems pass through an internal resonance. The existence of such a phenomenon is not restricted to coupled pendulums, but is inherent to a wide class of both linear and nonlinear parametric oscillatory systems. This opens up the possibility of designing new types of energy traps and absorbers for the dynamic protection of various mechanical systems.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction

Since a new principle of energy trapping based on the phenomenon of nonlinear resonance was discovered [1–4], the problem of the targeted energy transfer (TET) continues to attract a growing attention [5]. It was shown [6] that nonlinear resonance-based energy trapping is strongly connected with the existence of Limiting Phase Trajectories (LPTs), which have been introduced in [7] as an alternative to normal modes to describe the complete energy exchange between a protected system and an essentially nonlinear energy trap. In the case of TET, the existence of an LPT ensures the unidirectional energy transfer to the trap, the inverse energy flow being essentially forbidden, due to the strong nonlinearity of the trap and the damping. Therefore, it would seem that a 'linear' energy trap should not exist. However, there is a well-known generic example of a linear quantum phenomenon, namely Landau-Zener tunneling (LZT), in which a quantum system tunnels across an energy gap between two anti-crossed energy levels [8–10]. Quantum LZT was observed in semiconductor superlattices for electrons [11,12], as well as in optical lattices for ultracold atoms [13,14] and Bose–Einstein condensates [15,16]. The external forcing responsible for nonadiabatic energy-level crossing and LZT is exerted by an external electric field in the case of electrons in a semiconductor. Gravitation or acceleration fields play a similar role for ultracold atoms and Bose–Einstein condensates. Landau-Zener tunneling of optical waves was observed in optical lattices [17] and optical waveguide arrays [18]. Recently, Landau-Zener tunneling of acoustic waves in ultrasonic superlattices was predicted and observed [19]. In addition, these predictions were also extended to and observations made in 2D phononic crystals made of rigid cylinders in water [20,21]. An effective external forcing in optical or acoustic LZT is produced by perturbations of the corresponding optical or ultrasonic lattice.

The common feature of the aforementioned examples of nonadiabatic LZT is the irreversible (and almost unidirectional) exchange of energy between two states caused by an external forcing or perturbation. This type of exchange is desirable in vibrating mechanical systems, in which an impact excitation threatening the structural integrity of the system must be irreversibly transferred to a sacrificial subsystem. It turns out that such a classical system, governed by equations similar to those of a quantum system, can in fact be designed. We have pointed out earlier [22] a profound analogy between *adiabatic* quantum tunneling and energy exchange between weakly coupled classical linear oscillators. In this paper, we present for the first time a vibration analog of *nonadiabatic* Landau-Zener tunneling, which opens up the possibility of designing new types of energy traps.

2. Theory

We consider a system of two linear pendulums with lengths l_1 and l_2 and masses m_1 and m_2 weakly coupled by a spring.

* Corresponding author. Tel.: +1 217 333 3048; fax: +1 217 244 6534.
E-mail address: avakakis@illinois.edu (A.F. Vakakis).

0020-7462/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijnonlinmec.2010.08.010