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Revisiting umbra-Lagrangian–Hamiltonian mechanics: Its variational foundation and extension of Noether's theorem and Poincare–Cartan integral

Amalendu Mukherjee, Vikas Rastogi, Anirvan Dasgupta *

Mechanical Engineering Department, Indian Institute of Technology, Kharagpur 721302, India

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ABSTRACT

This paper revisits an extension of the Lagrangian–Hamiltonian mechanics that incorporates dissipative and non-potential fields, and non-integrable constraints in a compact form, such that one may obtain invariants of motion or possible invariant trajectories through an extension of Noether's theorem. A new concept of umbra-time has been introduced for this extension. This leads to a new form of equation, which is termed as the umbra-Lagrange's equation. The underlying variational principle, which is based on a recursive minimization of functionals, is presented. The introduction of the concept of umbra-time extends the classical manifold over which the system evolves. An extension of the Lagrangian–Hamiltonian mechanics over vector fields in this extended space has been presented. The idea of umbra time is then carried forward to propose the basic concept of umbra-Hamiltonian, which is used along with the extended Noether's theorem to provide an insight into the dynamics of systems with symmetries. Gauge functions for umbra-Lagrangian are also introduced. Extension of the Poincare–Cartan integral for the umbra-Lagrangian theory is also proposed, and its implications have been discussed. Several examples are presented to illustrate all these concepts.

1. Introduction

The use of Lagrange's equation formulated by Lagrange [1] offers the quickest way of deriving system equations for complex physical systems. However, it suffers heavily due to its inherent limitations in dealing with non-conservative and non-potential fields. Rayleigh [2] added velocity dependent potential in this equation but it affects the structural uniformity of the equation. Non-holonomic or non-integrable constraints are introduced through Lagrange's undetermined multipliers. Gyroscopic forces also require some special considerations.

Several papers have appeared that incorporate non-conservative and dissipative forces in the Lagrangian formulation with variational method, and show the connection between symmetries and existence of conserved quantities, thus finding invariants of motion of dynamical systems. Some significant works in this area were reported by Vujanovic [3–5] and Djukic [6,7] in their several papers. Vujanovic [4] had formulated a method for finding the conserved quantities of non-conservative holonomic system, which is based on the differential variational principle of d'Alembert. Using the similar method, Bahar and Kwatny [8] extended this idea to Noether's theorem [9] for constrained, non-conservative dynamical systems, which includes the influence of dissipation and constraints. Some other useful results related with the symmetry aspects of Lagrangian and Hamiltonian formalism are contained in papers of Katzin and Levine [10] and Sarlet and Cantrijn [11]. Recently, Simic [12] analyzed polynomial conservation laws of one-dimensional non-autonomous Lagrangian dynamical system.

Apart from all previous approaches, Mukherjee [13,14,18] suggested an alternative method of finding invariants of motion or possible invariant trajectories for dynamical systems. In his new method, a modified version of Lagrange's equation [13] was proposed through introduction of an additional time like variable called 'umbra-time', and this notion was extended to all types of energies as well as the Lagrangian itself. Mukherjee and Samantaray [14] introduced a step-by-step approach to arrive at umbra Lagrangian through system bondgraphs [15,16] and extending the idea of Karnopp's algorithm [17] to a broader class of systems. In continuation, Mukherjee [18] consolidated this idea and presented an important issue of invariants of motion for a general class of symmetric system through the concept of umbra-Lagrangian and an extension of Noether's theorem. Recently, an extension of this theory to continuous systems has been proposed, and the dynamics of an internally damped rotor driven through dissipative coupling has been studied [19].

This paper presents the aforementioned developments in an organized manner and elaborates the underlying variational or least action doctrine leading to the proposed form of equation for a general class of system. A detailed outline of its underlying

* Corresponding author. Tel.: +91 3222 282946; fax: +91 3222 282278.
E-mail addresses: amalendu@mech.iitkgp.ernet.in (A. Mukherjee), rastogivikas@yahoo.com (V. Rastogi), anir@mech.iitkgp.ernet.in (A. Dasgupta).

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