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Non-linear convection due to compositional and thermal buoyancy in Earth's outer core

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ABSTRACT

The linear stability of convection due to compositional and thermal buoyancy in Earth's outer core has been investigated. We have obtained the values of Takens–Bogdanov bifurcation points by plotting graphs of neutral curves corresponding to stationary and oscillatory convection for different values of physical parameters. We have derived a non-linear two-dimensional Landau–Ginzburg equation with real coefficients near the onset of stationary convection at a supercritical pitchfork bifurcation and two non-linear one-dimensional coupled Landau–Ginzburg type equations with complex coefficients near the onset of oscillatory convection at a supercritical Hopf bifurcation. We have studied Nusselt number contribution from a Landau–Ginzburg equation at the onset of stationary convection. We have discussed the stability regions of standing and travelling waves. We have also discussed the occurrence of secondary instabilities such as Eckhaus, zigzag and Benjamin–Feir instabilities. We have also derived the non-linear amplitude equation near the Takens–Bogdanov bifurcation point.

1. Introduction

Convection due to compositional and thermal buoyancy in a medium uniformly heated from below is of considerable interest in geophysical fluid dynamics, as this phenomena occurs within the Earth's outer core. In the case of the outer core, compositional and thermal buoyancy are produced at the inner core as the inner core grows [1]. Earth's outer core consists of molten iron and lighter alloying element, sulphur, in its molten form. This lighter alloying element present in the liquid phase is released as new iron freezes due to supercooling onto the solid inner core. Hence we get mushy layer near the inner core boundary where the problem becomes convective instability in a porous medium [2]. In mushy layer we will have solid iron particles, iron and sulphur in liquid form. In this paper we consider convective instability in a non-porous medium where lower layer is slightly above the mushy layer so that both iron and sulphur are in liquid phase and we take the upper layer as core-mantle boundary.

The mechanism of convection due to compositional and thermal buoyancy is similar to thermohaline convection except for the fact that a temperature difference can drive a mass current. Owing to the two-component nature of the fluid in outer core one has Soret effect and this leads to an additional control parameter ψ (separation ratio) besides thermal Rayleigh number R. Convection due to compositional and thermal buoyancy in two-component fluid is capable of showing stationary convection at pitchfork bifurcation, oscillatory convection at Hopf bifurcation and stationary convection at Takens–Bogdanov bifurcation corresponding to double zero eigenvalue. Takens–Bogdanov bifurcation point is the one at which the neutral curve of oscillatory convection intersects the neutral curve of stationary convection and the frequency on the neutral curve of oscillatory convection approaches to zero. This Takens–Bogdanov bifurcation point (where Rayleigh number for the oscillatory convection coincides with Rayleigh number for the stationary convection at the same wave number) is different from the codimension-two bifurcation point (where the Rayleigh number for the onset of stationary convection coincides with the Rayleigh number for the onset of oscillatory convection but normally at different wave numbers). Thus a codimension-two bifurcation point occurs at a point in parameter space where two curves having points corresponding to onset of stationary convection and oscillatory convection intersects, thus leading to a competition between these two types of onset of convection. However, for the problem of thermohaline convection and convection due to compositional and thermal buoyancy with stress-free boundaries,

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