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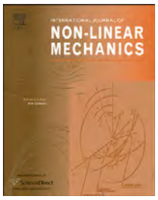

On a pure complementary energy principle and a force-based finite element formulation for non-linear elastic cables

H.A.F.A. Santos [a],*, C.I. Almeida Paulo [b]

[a] *Institute for Computational Engineering and Sciences, The University of Texas at Austin, 201 East 24th Street, 1 University Station C0200, Austin, TX 78712, USA*
[b] *Departamento de Engenharia Civil e Arquitectura, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal*

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A complementary-dual force-based finite element formulation is proposed for the geometrically exact quasi-static analysis of one-dimensional hyperelastic perfectly flexible cables lying in the two-dimensional space. This formulation employs as approximate functions the exact statically admissible force fields, *i.e.*, those that satisfy the equilibrium differential equations in strong form, as well as the equilibrium boundary conditions. The formulation relies on a principle of total complementary energy only expressed in terms of force fields, being therefore called a pure principle. Under the assumption of stress-unilateral behavior, this principle can be regarded as being dual to the principle of minimum total potential energy, corresponding therefore to a maximum principle. Some numerical applications, including cables suspended from two and three points at the same level or at different levels, with both Hookean and Neo-Hookean material behaviors, are presented. As it will be shown, in contrast to the standard two-node displacement-based formulation derived from the principle of minimum total potential energy, the proposed dual force-based formulation is capable of providing the exact solution of a given problem only using a single finite element per cable. Both the proposed principle of pure complementary energy and its corresponding force-based finite element formulation can be easily extended to the case of cables lying in the three-dimensional space.

1. Introduction

Cables are used in many engineering applications, such as suspension bridges, cable-stay bridges, suspension roofs, aerial tramways, aerospace deployable structures, biomechanical systems such as biological tissues and cytoskeleton, etc.

Exhibiting a highly geometrically non-linear behavior, cable structures are very flexible and undergo large displacements before attaining their equilibrium configuration. As a consequence, the effects of large displacements need to be considered when establishing the equilibrium equations of the cable. A brief review of the early history of the research on the behavior of cables has been published in [11].

Cable structures may be divided into two categories, according to their loading: (i) cables supporting concentrated loads and (ii) cables supporting distributed loads. Although all the results presented in this work are valid for both categories, we are mainly concerned with cables belonging to the second category, since the hypothesis of loads acting only at certain points leads to simple algebraic formulations.

There are usually two major approaches for the development of cable finite elements, namely, the exact analytical method and the displacement-based method. In the former approach the cable element is derived from analytical expressions of the deformed geometry of the cable which, in a mathematical sense, exactly describe the cable under certain load conditions [20,23,12,15,27]. The latter approach is very often based on Lagrangian functions for the interpolation of the geometry of the cable element. The two-node straight bar element is the most common element used in the modeling of cables. Several displacement-based finite element formulations have been presented for the geometrically non-linear analysis of cables, see for example [2,5]. A drawback of these elements is the spurious slope discontinuities occurring at nodes where no concentrated loads act. These discontinuities are due to the straight element assumption and may lead to convergence problems in the analysis. Less common are the three [1] and four-node [18] elements, which use parabolic and cubic interpolation functions, respectively. Also both geometrically and physically non-linear finite element analysis employing the displacement-based method have been proposed to model non-linear elastic cable problems [10].

Although displacement-based formulations may lead to sufficiently accurate displacement fields, the corresponding stress-resultant fields may be highly erroneous. This occurs since the

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* Corresponding author.
E-mail address: hugofreixialsantos@gmail.com (H.A.F.A. Santos).

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