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The analysis of the spectrum of Lyapunov exponents in a two-degree-of-freedom vibro-impact system

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ABSTRACT

In the study of dynamical systems, the spectrum of Lyapunov exponents has been shown to be an efficient tool for analyzing periodic motions and chaos. So far, different calculating methods of Lyapunov exponents have been proposed. Recently, a new method using local mappings was given to compute the Lyapunov exponents in non-smooth dynamical systems. By the help of this method and the coordinates transformation proposed in this paper, we investigate a two-degree-of-freedom vibro-impact system with two components. For this concrete model, we construct the local mappings and the Poincaré mapping which are used to describe the algorithm for calculating the spectrum of Lyapunov exponents. The spectra of Lyapunov exponents for periodic motions and chaos are computed by the presented method. Moreover, the largest Lyapunov exponents are calculated in a large parameter range for the studied system. Numerical simulations show the success of the improved method in a kind of two-degree-of-freedom vibro-impact systems.

1. Introduction

The phenomena of vibro-impact often appear in many engineering fields such as mechanism, vehicle, etc. The research of the vibro-impact problems is significant in these fields. Many scholars have widely analyzed the dynamical behaviors of the vibro-impact systems. To a simple mathematical model for the bouncing ball, Shaw and Holmes [1] proved the existence of period-doubling bifurcation cascade and found the Smale horseshoe in the system. Nordmark [2] analyzed the grazing bifurcation by using the method of local mapping, and found that the grazing bifurcation was the main cause of the transition from periodic motions to chaotic motions. Lamba and Budd [3] discovered that the jumping of the largest Lyapunov exponents of non-smooth dynamical system followed by changing of parameters in grazing bifurcation. By using the center manifold and normal form method, Luo et al. [4] considered local codimension two bifurcation of mappings in the two-degree-of-freedom vibro-impact system. Bishop et al. [5] investigated an impacting driven beam, which was suppressed from chaotic motions to a period-1 orbit. Pavlovskaia and Wiercigroch [6] proposed a graphical method of iteration of the two-dimensional mapping that similar to the cobweb method used in the one-dimensional mapping for an impact oscillator with drift. Dankowicz and Zhao [7] discussed the local behaviors of codimension one and codimension two grazing bifurcations by using the discontinuity mappings in an impact microactuator.

The spectrum of Lyapunov exponents is one of the main tools for estimating the stability and chaos of dynamical systems. It denotes the numerical character of average exponent divergence or convergence rate of the neighboring trajectory in the phase space [8,15]. The investigation of the spectrum of Lyapunov exponents of smooth dynamical systems is relatively perfect at present. But the method for calculating the spectrum of Lyapunov exponents of smooth dynamical systems cannot be applied to non-smooth dynamical systems directly. So far, some researchers have come up with some calculation methods of the spectrum of Lyapunov exponents in some specific non-smooth dynamical systems. Stefanski et al. [9,10] investigated the largest Lyapunov exponent for mechanical systems with impact using the properties of synchronization phenomenon. Supplementing certain transitional conditions to the linearized equations at the instants of impacts, Müller [11] applied the classical calculation methods of the spectrum of Lyapunov exponents of smooth dynamical systems to non-smooth dynamical systems. de Souza and Caldas [12] introduced the transcendental mappings to describe the solutions of integrable differential equations between impacts, supplemented by transitions at the instants of impacts. Hence, the methods for calculating the spectrum of Lyapunov exponents of smooth discrete dynamical systems could be used to non-smooth systems. Jin and Lu [13,14] focused on the local mappings of non-smooth systems and found a general calculation method of the spectrum of

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