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Forced large amplitude periodic vibrations of non-linear Mathieu resonators for microgyroscope applications

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ABSTRACT

This paper describes a comprehensive non-linear multiphysics model based on the Euler–Bernoulli beam equation that remains valid up to large displacements in the case of electrostatically actuated Mathieu resonators. This purely analytical model takes into account the fringing field effects and is used to track the periodic motions of the sensing parts in resonant microgyroscopes. Several parametric analyses are presented in order to investigate the effect of the proof mass frequency on the bifurcation topology. The model shows that the optimal sensitivity is reached for resonant microgyroscopes designed with sensing frequency four times faster than the actuation one.

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1. Introduction

Microelectromechanical gyroscopes have attracted a lot of attention due to their small size, batch fabrication, Integrated Circuit (IC) compatibility, low cost, and acceptable moderate performance for most applications. They can be used either as a low-cost miniature companion with micromachined accelerometers to provide heading information for inertial navigation purposes or in other areas, including ride stabilization and roll-over detection, video-camera stabilization, virtual reality, inertial mouse for computers, robotics applications, and a wide range of military applications [1,2].

Micromachined gyroscopes typically rely on energy transfer between two orthogonal vibration modes through the Coriolis acceleration, based on the combination of vibration of a proof-mass and an orthogonal angular-rate input. As shown in Fig. 1, when the gyroscope is subjected to an angular rotation, a sinusoidal Coriolis force is induced in the direction orthogonal to the drive-mode oscillation at the driving frequency.

To achieve high sensitivity in conventional micro rate gyroscopes based on harmonic oscillators, the drive and the sense resonant frequencies are typically designed and tuned to match, and the device is controlled to operate at or near the peak of the response curve (whose amplitude is defined by the *Q*-factor) [3]. However, current micro fabrication processes produce asymmetries causing frequency mismatching between modes, translating to drastic loss of sensitivity [1]. Although solutions to overcome frequency mismatching have been pursued [4,5], many of them involve adding complexity to the system by including additional controllers, additional degrees of freedom [6] or utilizing multiple drive mode oscillators [7].

Due to the complexity of the proposed control schemes [4,5], alternate approaches were considered. One of them consists in using the parametric resonance for the actuation in order to overcome the loss of sensitivity issue [8] and improve the performances of vibrating microgyroscopes [9]. Another alternative is the resonant sensing [10] of the Coriolis force instead of displacement sensing employed in most conventional microgyroscopes. Taking advantage of the high sensitivity of the resonant detection, the matching of the drive and the sense frequencies is not mandatory to achieve a high resolution. Consequently, the number of states that have to be simultaneously controlled and the number of variables that require identification are much smaller and the dynamics is simplified from a minimally two-dimensional system to a series of coupled one-dimensional mass–spring–damper systems. Nevertheless, resonant sensing suffers from the early occurrence of non-linearities [11–14] which reduces the gyroscope performances. Moreover and unlike resonant accelerometers, the sensing parts are Mathieu resonators

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