International Journal of Non-Linear Mechanics 46 (2011) 125–141

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Stability and non-linear second-order elastic analyses of beam and framed structures with semi-rigid connections using the cross method

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ARTICLE INFO

Article history:
Received 6 May 2008
Received in revised form
27 July 2010
Accepted 29 July 2010

Keywords:
Beams
Beam-columns
Large deflections
Non-linear analysis
Semi-rigid connections
Second-order analysis
Stability
Structural analysis

ABSTRACT

The main objective of this publication is to present an extended version of the Moment Distribution Method (MDM) for the stability and non-linear second-order analysis of indeterminate beams and framed structures made of beam-columns of symmetrical cross-section including the combined effects of shear and bending deformations, axial loads, and semi-rigid connections. The proposed method along each member has the following advantages: (1) it can be utilized in the first- and second-order analyses (including buckling analysis) of indeterminate beams and framed structures made of beam-columns with rigid, semi-rigid, and simple end connections; (2) the effects of semi-rigid connections are condensed into the bending stiffness and fixed-end moments without introducing additional degrees of freedom and equations of equilibrium; and (3) it is accurate, powerful, practical, versatile, and an excellent teaching tool. Analytical studies indicate that shear deformations, semi-rigid connections, and axial loads increase the lateral deflections and affect the internal moments and reactions of continuous beams and framed structures. These effects must be taken into account particularly in slender structures and when they are made of beam or columns with relatively low effective shear areas (like laced columns, columns with batten plates or with perforated cover plates, and columns with open webs) or with low shear stiffness (like short columns made of laminated composites with low shear modulus G when compared to their elastic modulus E) making the shear stiffness GA_s of the same order of magnitude as EI/L^2. These effects become even more significant when the external supports are not perfectly clamped. Three comprehensive examples are included that show the effectiveness of the proposed method.

1. Introduction

The slope-deflection method and the Hardy Cross method represent the starting points in the evolution and development of the matrix stiffness method as it is known today [1]. The slope-deflection method was presented in 1915 by Wilson and Maney [2] in a *Bulletin* from the University of Illinois at Urbana-Champaign as a general method to be used in the analysis of beam structures with rigid joints subjected to transverse loads. Later, Hardy Cross [3,4] also from the University of Illinois at Urbana-Champaign proposed the first numerical method used in the structural analysis of indeterminate rigid frames that he called “the moment distribution method”. The great merit of the Cross method is that it made possible the efficient and safe design of many buildings and rigid jointed frames over half a century [5,6].

In the Cross method it is assumed that the rigid joints of frame members are initially fixed against rotation. The fixed-end moments produced by external loads are computed first as well as the distribution and carry-over factors of each member. These fixed-end moments are unbalanced at the joints of the original non-restrained structure. In order to have rotational equilibrium at each joint, the moment is distributed proportionally to the corresponding member stiffness. These distributed moments are associated with the so-called “carry-over” moments at the opposite ends of structural members. They are considered to be new incremental unbalanced moments and the procedure repeats until the unbalanced moments become negligible. The true moments at the ends of all members are the sum of all distributed moment increments. Initially, the Cross method assumes joint rotations only. However, for frames with joint translations a more general scheme is used, which requires the application of the method successively by setting up a system of equations with the joint translations or sways as unknowns.

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doi:10.1016/j.ijnonlinmec.2010.07.009