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On the global dynamics of chatter in the orthogonal cutting model

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ABSTRACT

The large-amplitude motions of a one degree-of-freedom model of orthogonal cutting are analysed. The model takes the form of a delay differential equation which is non-smooth at the instant at which the tool loses contact with the workpiece, and which is coupled to an algebraic equation that stores the profile of the cut surface whilst the tool is not in contact. This system is approximated by a smooth delay differential equation without algebraic effects which is analysed with numerical continuation software. The grazing bifurcation that defines the onset of chattering motion is thus analysed as are secondary (period-doubling, etc.) bifurcations of chattering orbits, and convergence of the bifurcation diagrams is established in the vanishing limit of the smoothing parameters. The bifurcation diagrams of the smoothed system are then compared with initial value simulations of the full non-smooth delay differential algebraic equation. These simulations mostly validate the smoothing technique and show in detail how chaotic chattering dynamics emerge from the non-smooth bifurcations of periodic orbits.

1. Introduction

In the machining industry it is well known that at high material removal rates, turning, milling and drilling processes are subject to chattering motions in which the tool repeatedly loses and re-establishes contact with the workpiece. Chattering results in a very poor quality finish, see Fig. 1(a), so in practice the need to avoid it limits the material removal rate and places bounds on the technological parameters such as the spindle speed and the depth of cut (also known as the chip width). These bounds are in addition to those due to the power and torque characteristics of the machine.

The general aim of this paper is to understand in more detail the dynamical mechanisms involved in the onset of chattering in turning processes, where a quasi-stationary tool cuts into a rapidly rotating workpiece. To simplify matters, we restrict our analysis to the special case of *orthogonal cutting*, depicted in Fig. 2.

In turning processes, the cutting force between the tool and the workpiece is a function of the chip thickness, that is, the difference between the tool position and the surface position. The surface position was determined one (or more) revolutions earlier by the tool's past motion. Turning processes are thus *regenerative* [1,2] in the sense that the tool's motion is forced by its own history, leading to models based on delay differential equations (DDEs). These models admit equilibria corresponding to steady cutting, where the chip thickness is independent of time, but the stability of this regime is not assured. By analysing when steady cutting is linearly unstable, we may thus identify combinations of technological parameters which the machinist should avoid [3–10].

In practice, the stability of the process is complicated by non-linear effects, and the typical situation is indicated by the one-dimensional bifurcation diagram in Fig. 1(b), where we fix the spindle speed and analyse the dynamics of the tool as the chip width is varied. Here $\|x\| := \max_t x(t) - \min_t x(t)$ denotes the peak-to-peak magnitude of the tool's motion $x(t)$, so that on branch A, where $\|x\| = 0$, we have linearly stable steady cutting in which the chip thickness is constant. As the chip width is increased, steady operation becomes linearly unstable at the Hopf bifurcation point B. However, normal form computations [11,12] and measurements [13,14] show that the bifurcation is subcritical, indeed robustly so over a wide range of parameters and cutting force characteristics [15]. Hence the branch C of periodic orbits that emanates 'bends back' and the periodic orbits on it are themselves unstable and hence are not observable in experiments.

However, we know from experiments [13,16] that complicated large amplitude chattering motions E (Fig. 1(b)) are possible at depths of cut less than that of the Hopf bifurcation point. Hence in regime 2, there is bistability in that sufficiently large perturbations to steady cutting can result in sustained chattering oscillations [13]. In contrast, in regime 1, only steady cutting is possible and in

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