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Non-linear modal analysis of a rotating beam

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The free non-linear vibration of a rotating beam has been considered in this paper. The von Karman strain–displacement relations are implemented. Non-linear equations of motion are obtained by Hamilton's principle. Results are obtained by applying the method of multiple scales to a set of discretized ordinary differential equations which obtained by using the Galerkin discretization method. This set contains coupling between transverse and axial displacements as quadratic and cubic geometric non-linearities. Non-linear normal modes and non-linear natural frequencies with or without internal resonance are observed. In the internal resonance case, the internal resonance between two transverse modes and between one transverse and one axial mode are explored. Obtained results in this study are compared with those obtained from literature. The stability and some dynamic characteristics of the non-linear normal modes such as the phase portrait, Poincare section and power spectrum diagrams have been inspected. It is shown that, for the first internal resonance case, the beam has one stable or degenerate uncoupled mode and either: (a) one stable coupled mode, (b) one unstable coupled mode, (c) two stable and one unstable coupled modes, (d) three stable coupled modes, and (e) one stable coupled mode. On the other hand, for the second internal resonance case, the beam has one stable or unstable or degenerate uncoupled mode and either: (a) two stable coupled modes, (b) two unstable coupled modes, and (c) one stable coupled mode depending on the parameters.

1. Introduction

The vibration investigation of a rotating beam which may be a simplified model of a rotating structure such as turbine blades, robot manipulators, and helicopter blades has significances in the efficiency and design of these structures. The survey of free vibration analysis of these structures such as calculation of their non-linear normal modes (NNMs) and their non-linear natural frequencies (NNFs) is a crucial goal of researchers and designers.

The definition of the NNMs was first presented by Rosenberg [1]. Then many researchers used this definition to construct NNMs in different ways. Shaw and Pierre [2] presented a practical methodology for deriving the normal modes of motion for continuous media based on invariant manifold theory. They obtained NNMs and NNFs of three simply supported beams without internal resonance using this approach. King and Vakakis [3] expressed an energy-based method for constructing the NNMs of discrete conservative systems. Nayfeh and Nayfeh [4,5] used the method of multiple scales (MMS) for investigation of the NNMs of one-dimensional continuous systems with or without internal resonance, with weak cubic inertial and geometrical non-linearities and with quadratic and cubic geometrical non-linearities, respectively. Mazzilli et al. [6] constructed the NNMs of an axially loaded beam by using the MMS. They verified the results such as time histories, phase portraits and mode shapes with the outcomes from an invariant manifold solution.

The next important aspects of the NNMs are their stability and bifurcation. Nayfeh et al. [7] examined NNMs of a fixed–fixed buckled beam with internal resonance by applying the MMS directly to the equations of motion. They also explored the stability and bifurcations of the NNMs. Lacarbonara et al. [8] studied the construction of resonant NNMs of a generic group of weakly non-linear one-dimensional continuous systems with quadratic and cubic geometric non-linearities, employing the MMS for the cases of 2:1, 1:1, and 3:1 internal resonances. They also surveyed the stability and orthogonality of NNMs. Li et al. [9] investigated the NNMs and their bifurcations of a complex two DOF system with both quadratic and cubic non-linearities with or without internal resonance by applying the MMS.

Most researches on rotating beams have concentrated on implementing invariant manifold theory for construction of NNMs. Crespo da Silve and Hodges [10] formulated a general rotating beam which had a pre-cone angle and a variable pitch angle, with

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