International Journal of Non-Linear Mechanics 46 (2011) 819–827

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

On Maxwell fluids with relaxation time and viscosity depending on the pressure

Satish Karra [a], Vít Průša [b,1], K.R. Rajagopal [a,*]

[a] Texas A&M University, Department of Mechanical Engineering, 3123 TAMU, College Station, TX 77843-3123, United States
[b] Faculty of Mathematics and Physics, Charles University in Prague, Sokolovská 83, Praha 8 – Karlín CZ 186 75, Czech Republic

ARTICLE INFO

Article history:
Received 7 May 2010
Accepted 28 February 2011
Available online 9 March 2011

Keywords:
Maxwell fluid
Pressure dependent material moduli
Stokes' second problem
Exact solution

ABSTRACT

We study a variant of the well-known Maxwell model for viscoelastic fluids, namely we consider a Maxwell fluid with viscosity and relaxation time depending on the pressure. Such a model is relevant for example in modelling the behaviour of some polymers and geomaterials. Although it is experimentally known that the material moduli of some viscoelastic fluids can depend on the pressure, most of the studies concerning the motion of viscoelastic fluids do not take such effects into account despite their possible practical significance in technological applications. Using a generalized Maxwell model with pressure dependent material moduli we solve a simple boundary value problem and we demonstrate interesting non-classical features exhibited by the model.

1. Introduction

The Maxwell fluid model was originally developed by Maxwell [1] to describe the elastic and viscous response of air. Nowadays, it is however, frequently used to model the response of various viscoelastic fluids ranging from polymers—see for example Ferry [2]—to the Earth's mantle—see for example Cathles [3]. In the present paper, we study an important generalization of the original model due to Maxwell, namely we consider a model with pressure dependent material moduli. While the model with a pressure dependent material moduli has important technological ramifications, little is known about the qualitative or quantitative features related to the model. In fact, there is no careful analytical study with regard to mathematical questions concerning existence and uniqueness of solutions for such fluids. Even within the context of solutions to initial-boundary value problems, there is no systematic study when all the material moduli are pressure dependent. Given the possible usefulness of this model, it is surprising that there are no such studies and the analysis carried out here addresses this lacuna.

Bridgman [4] in his pioneering experiments in high pressure physics reported (for many organic fluids) a significant dependence of the viscosity on the pressure and articulated the need for the dependence of the material moduli on the pressure. Models of fluids with pressure dependent viscosity are nowadays frequently used to describe the behaviour of fluids in many applications, for example in lubrication theory, see Neale [5] and Gwynllyw et al. [6]. With regard to viscoelastic fluids, we can ask whether the material moduli of viscoelastic fluid models also exhibit dependence on pressure, and what effects can be described if we use pressure dependent material moduli. Viscoelastic material moduli depending on the pressure has been reported for polymers, see for example Singh and Nolle [7], McKinney and Belcher [8] and the literature stemming from these papers, as well as for geomaterials, see for example Weertman et al. [9], Ivins et al. [10] and Sahaphol and Miura [11].

The question on pressure dependent viscosity and/or relaxation time is especially interesting, for example, with respect to applications in geophysics, since the material of the Earth's mantle is subject to a wide range of pressures. If we consider a material stratified due to the influence of the gravitational force, then because of pressure dependent material moduli, we would be dealing with a body with material moduli depending on the vertical coordinate. A similar situation occurs in geophysical applications and usually it is assumed that the body (in this case the Earth's mantle) is composed of a number of layers of materials with constant material moduli—a paradigm introduced in the papers by McConnel [12,13]. The approach based on material moduli dependence on the pressure and consequently (in the case of a stratified material) on the vertical coordinate provides an alternative approach to the problem. In such an approach, the material moduli continuously vary with the depth, in contrast to the assumptions of McConnel [13]. Although geophysicists and polymer engineers are aware of the possibility of pressure dependent material moduli, this dependence is invariably ignored in studies concerning the dynamics of these materials. If we take

* Corresponding author.
E-mail addresses: satkarra@tamu.edu (S. Karra), prusv@karlin.mff.cuni.cz (V. Průša), krajagopal@tamu.edu (K.R. Rajagopal).

[1] Current address: Texas A&M University, Department of Mechanical Engineering, 3123 TAMU, College Station, TX 77843-3123, United States.

0020-7462/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijnonlinmec.2011.02.013