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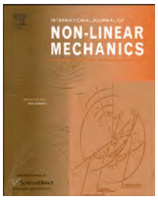

Exceedance probability criterion based stochastic optimal polynomial control of Duffing oscillators

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ABSTRACT

An optimal polynomial control strategy is developed in the context of the physical stochastic optimal control scheme of structures that is well-adapted to randomly-driven non-linear dynamical systems. A class of Duffing oscillators with polynomial active tendons subjected to random ground motions is investigated for illustrative purposes. Numerical studies reveal that using an exceedance probability criterion with the minimum of the failure probability of system quantities in energy trade-off sense, a linear control with the 1st-order controller suffices even for strongly non-linear systems. This bypasses the need to utilize non-linear controls with the higher-order controller which may be associated with dynamical instabilities due to time delay and computational dynamics. The statistical variability, meanwhile, of system responses gains an obvious reduction, and the system performance is significantly improved. The 1st-order controller, however, does not have the same control effect to the higher-order controller when control criteria currently in used are employed, e.g. system second-order statistics evaluation and Lyapunov asymptotic stability condition, as indicated in the comparative studies of the exceedance probability criterion against the two control criteria. Besides, the proposed optimal polynomial control is insensitive to the non-linearity strength of the class of base-excited non-linear oscillators whereby a robust control of systems can be implemented, while the LQG control in conjunction with the statistical linearization technique, using a band-limited white noise input, does not have this advantage.

1. Introduction

Classical stochastic optimal controls exclusively hinge on Itô-type stochastic differential equations, where the random disturbance term specifying external excitations and measurement noises is mathematically assumed to be independent additive Gaussian white noise or filtered Gaussian white noise [1]. The well-developed stochastic optimal control method, i.e. linear quadratic Gaussian (LQG) control [2], and its derivative control criteria for designing optimal control gain, such as the minimum variance control criterion [3], neighboring optimal control criterion [4], probability density function (PDF) tracing control criterion [5], optimal feedback control criterion based on the stochastic averaging method for quasi-Hamiltonian systems and the Hamilton–Jacobi–Bellman (HJB) equation [6] and reliability-based control criterion [7], are all carried out in the context of this theoretical framework. The applicability of the above canonical theory in the stochastic optimal control of structures, however, still remains an open challenge to the practical non-stationary, non-Gaussian white noise driven system, e.g. earthquake ground motions, strong winds and sea waves usually encountered in civil engineering. As an insight into this challenge, a physical approach to structural stochastic optimal controls, on the basis of the generalized density evolution equation, has been proposed, which is applicable to stochastic dynamical systems subjected to practical random excitations. For illustrative purposes, two cases of linear stochastic dynamical systems were investigated to prove the validity and applicability of the physical methodology [8]. In the present paper, we address the stochastic optimal control of randomly-driven non-linear systems in the context of this physical scheme. Specifically, a class of base-excited Duffing oscillators is investigated. The Duffing oscillators, in fact, provide a useful model for a family of physical non-linear dynamical systems with behaviors of either hardening or softening [9,10].

The pioneered investigation of optimal controls of non-linear dynamical systems was carried out in terms of a suboptimal solution of the HJB equation for a non-quadratic cost function [11]. Since then, many fruitful non-linear optimal control methods have been proposed and studied, e.g. pulse control [12], instantaneous optimal control [13], perturbation method [14], numerical optimal control [15], optimal polynomial control [16], sliding-mode control [17], acceleration control [18], generalized optimal control [19], neural network method [20]. Among these control strategies, the optimal

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