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## Effect of electromagnetic actuations on the dynamics of a harmonically excited cantilever beam

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## ABSTRACT

The influence of electromagnetic actuators (EMAs) on the frequency response of a harmonically excited cantilever beam is investigated analytically, numerically and experimentally in this paper. Specifically, the intensity of the current generating the EMAs force is varied and its effect on the dynamic behavior of the system is analyzed. Analytical treatment based on perturbation analysis is performed on a simplified equation modeling the one mode vibration of the cantilever beam. Results indicated that EMAs produce a softening behavior in the system. Further, it is shown that as the current intensity of EMAs increases, the resonance curve shifts toward smaller values of frequency and the non-linear characteristic of the system becomes softer. The analytical predictions have been verified numerically and confirmed experimentally using a test rig.

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## 1. Introduction

Non-linear behavior of mechanical systems can manifest itself in various forms [1,2]. For instance, in the automotive sector, the lateral vibrations of drive systems by belts may lead to a Duffing oscillator with parametric excitations [3]. Reducing this type of behavior requires an understanding of basic phenomena from analytical and experimental viewpoint. Therefore, it is of great importance to develop a strategy for reducing vibrations with large amplitudes, determine stability criteria and controlling nonlinear phenomena leading to large amplitude oscillations and hysteresis in the systems. Recent analytical results have shown that the introduction of a fast harmonic excitation in pendulumlike systems can have an effect on the elimination of hysteresis [4,5]. On the other hand, active magnetic bearings (AMBs) have proved their effectiveness in many industrial applications; they have the advantage to operate without contact and can be used in applications requiring clean or corrosive environments [6]. AMBs also have the advantage of being able to act on the shaft directly or indirectly by associating them with conventional bearings. In this case AMBs act as actuators [7,8].

In the same context, electromagnetic actuators, considered as simple means of excitation [9], can be exploited positively in industrial applications where attracting forces are needed. The

\* Corresponding author. E-mail address: jarir.mahfoud@insa-lyon.fr (J. Mahfoud). systems actuated by electromagnetic forces exhibit generally complicated behavior due to the non-linearities generated by the force. For instance, non-linear dynamics and chaos control for an electromagnetic system has been considered in [10], while active electromagnetic damping of lateral vibration of a cantilever beam, which is suitable for non-linear systems, has been investigated in [11].

The objective of this paper is to study analytically, numerically and experimentally, the influence of EMAs on the frequency response of an excited cantilever beam. Specifically, the effect of varying the intensity of the current generating the EMAs force on the dynamic behavior of the system is analyzed. The aim is to assess the possibility of tuning the first resonance frequency value of the structure by using actuators with constant current.

First, in order to capture basic phenomena and to qualify the effect of electromagnetic forces on the non-linear behavior, an analytical treatment of a one degree of freedom system consisting of a non-linear oscillator subjected to a periodic excitation is performed. The analytical treatment based on a perturbation analysis leads to an approximation of the amplitude–frequency response equation allowing the analysis of the influence of EMAs on the frequency response. Then, the numerical simulations by using finite element method were performed for several configurations as close as possible to the experiments. The objective is to examine the range of variation of the important parameters in order to choose the required air gap and current intensity leading to the suitable behavior. In order to validate the analytical and numerical predictions, experiments are realized using a test rig

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