Solution of Two-Dimensional Shallow Water Equations by TVD-MacCormack Scheme

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Abstract
In this paper, a TVD-MacCormack scheme for solution of two-dimensional shallow water equations has been presented. The scheme is hight-resolution second-order accurate which satisfies the well-known C-property over uneven bed topographies. The scheme is simple, very efficient and shock capturing as well. Although the accuracy is slightly reduced in comparison with other high-resolution schemes, the computational cost is low, since there is no need to compute eigenvalues and corresponding modal matrix. The results of the simulations have been compared with analytical solutions and other numerical methods and show satisfactory agreement.

Keywords: TVD-MacCormack, C-property, shallow water equations, Depth-averaged flow.

1. INTRODUCTION

Numerical simulations are now powerful tools for solving a wide range of problems in engineering. In many cases of hydraulic engineering problems, such as shallow water wave, flow in estuaries and so on, the flow can be properly modeled using two-dimensional depth-averaged free surface flow equations without taking complicated three-dimensional flow into account. Over the past two decades, researchers have applied two-dimensional depth-averaged computer models to a wide variety of hydraulic problems. Modeling of side channel flow, surges, flow in open channels and rivers, bends, flow around hydraulic structures, flow in transitions, dam break problem, and many other hydraulic problems have been successfully achieved by depth-averaged models. To effectively address real life problems, a numerical model should be able to capture shocks as well. In some circumstances, there may be discontinuities such as hydraulic jump in the domain of flow. Such discontinuities are referred to 'shocks' in the literature and the schemes which are able to predict these shocks are referred to 'shock capturing' methods. Indeed, shock capturing methods refer to a class of numerical methods for solving problems containing discontinuities (shocks, contact discontinuities, or other discontinuities), which can automatically ‘‘capture’’ these discontinuities without special effort to track them. This task is done by modern high-resolution schemes nowadays. More recently, high resolution schemes have been considerably developed to do this task as effective as possible. Flux-vector splitting (FVS), flux-difference splitting (FDS), essentially non-oscillating (ENO) schemes, weighted ENO (WENO) and total variation diminishing (TVD) schemes are such methods.

The FVS method decomposes the flux vector into positive and negative components according to the sign of eigenvalues of the flux Jacobian matrix. The FDS-type approaches use Godunov-type solutions that involve an approximate solution of the local Riemann problem.

In most of the aforementioned methods, the eigenvalues and corresponding matrix of eigenvectors should be evaluated in each time step, since the direction of waves depends on the sign and magnitude of eigenvalues of the flux Jacobian matrix. This process increases the computational costs dramatically. To reduce the computational costs, the well-known MacCormack scheme can be used to solve the governing equations of shallow water fluid flow in the absence of discontinuities or strong shocks. The MacCormack scheme is a two-step predictor-corrector method [1]. To enhance the MacCormack scheme to capture shocks and discontinuities, the total variation diminishing (TVD) approach has been combined with the MacCormack scheme and is called TVD-MacCormack method (e.g. [2, 3]). The method is very efficient and at the limited expense of a slight reduction in accuracy, the computational costs will reduce [4].