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A robust method to estimate suitable values for the parameters of general MQ-RBF to solve SWEs

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Abstract

Numerical simulation of Shallow Water Equations (SWEs) is an important requirement of hydraulic, river and coastal engineers for design purposes. Radial Basis Function based Differential Quadrature (RBF-DQ) is a meshfree and highly convergent method, but its widespread application is hampered by lacking a suitable scheme to objectively select the shape parameters incorporated in the RBFs. In this research, method of cross validation is used to estimate good values for the parameters and two numerical examples concerning with solving SWEs via RBF-DQ using general Multiquadric (MQ) RBF are carried out. The results show that the devised method is robust in estimating the parameters.

Keywords: Shallow water equations, Radial basis function, Differential quadrature, Multiquadric.

1. INTRODUCTION

Mathematical models based on Shallow Water Equations (SWEs) are considered as an important simulation tool for describing the hydrodynamic of flow in rivers, estuaries, and coastal areas. The current study concerns with practical implementation of a meshfree method so called Radial Basis Function-based Differential Quadrature (RBF-DQ) in the numerical simulation of hyperbolic two-dimensional SWEs. Among RBFs, generalized Multiquadric RBF (MQ-RBF) benefits from high exponential rate of convergence and excellent performance in capturing the variability in state variables over the spatial domain. However, widespread application of the radial basis functions in computational hydraulics is hampered by a lack of suitable mathematical theory to objectively select the free parameters incorporated in their definition. These free parameters are often referred to as the shape parameters whose numerical values greatly affect the convergence and accuracy of the approximation.

So far, many researches have been done in order to find an efficient method to estimate good values for the shape parameters. Some investigators stated that suitable values of the shape parameters are dependent on the number and distribution of nodes. They proposed some formulae for estimation of the parameters in terms of a new defined parameter which is an indicator of the number and distribution of nodes. As an example, for the parameter c in MQ-RBF, Hardy (1971) proposed the formula c=0.815d in which d is the average distance between each node to its nearest neighbor [1]. But this formula and other similar formulae cannot be useful for all problems, because, all of the factors affecting the value of the parameters (for example the function values at points) are not considered in such formulae. Rippa (1999) showed that the optimum value of a shape parameter which minimizes the error of interpolation is dependent on the number and distribution of nodes, the data vector and the precision of computation [2]. Some investigators devised methods to estimate the parameters by considering all of these factors. They used the method of cross validation to minimize the error of interpolation [2, 3]. However, this method has not become practical in all problems. Most of researchers prefer to assign values to the parameters by trial and error while solving differential equations.

In this research, it is shown again that the optimal values of these free parameters depend on the number and distribution of points, the data vector, and the precision of the computation. Besides, for applying RBF-DQ to solve SWEs, a methodology is proposed whereby the free parameters are optimized using cross validation. The developed methodology is evaluated by solving a simple linear system of SWEs having analytical solution. In addition, 2D SWEs are also solved with the same method and the results are compared with those obtained using MIKE 21 and POLCOMS (Proudman Oceanographic Laboratory Coastal Ocean Modelling System). MIKE 21 is a software simulating waterways in 2D and POLCOMS is a quasi-3D numerical model.