

# Locales: A Module System for Mathematical Theories

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**Abstract** Locales are a module system for managing theory hierarchies in a theorem prover through theory interpretation. They are available for the theorem prover Isabelle. In this paper, their semantics is defined in terms of local theories and morphisms. Locales aim at providing flexible means of extension and reuse. Theory modules (which are called locales) may be extended by definitions and theorems. Interpretation to Isabelle's global theories and proof contexts is possible via morphisms. Even the locale hierarchy may be changed if declared relations between locales do not adequately reflect logical relations, which are implied by the locales' specifications. By discussing their design and relating it to more commonly known structuring mechanisms of programming languages and provers, locales are made accessible to a wider audience beyond the users of Isabelle. The discussed mechanisms include ML-style functors, type classes and mixins (the latter are found in modern object-oriented languages).

**Keywords** Theorem prover · Module system · Theory hierarchy · Theory interpretation · Isabelle

## 1 Introduction

The developers of the computer algebra system Axiom pioneered implementing complex hierarchies of algebraic structures in a computer language. The user manual [13] shows a graph of 45 interconnected algebraic structures at 15 levels in the basic algebra hierarchy all of which are implemented as types in that system. Standard libraries of programming languages usually have many more classes, but hierarchies

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