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Generalising Unit-Refutation Completeness and SLUR via Nested Input Resolution

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Abstract The class $\mathcal{SLUR}$ (*Single Lookahead Unit Resolution*) was introduced in Schlipf et al. (Inf Process Lett 54:133–137, 1995) as an umbrella class for efficient (poly-time) SAT solving, with linear-time SAT decision, while the recognition problem was not considered. Čepek et al. (2012) and Balyo et al. (2012) extended this class in various ways to hierarchies covering all of CNF (all clause-sets). We introduce a hierarchy $\mathcal{SLUR}_k$ which we argue is the natural "limit" of such approaches. The second source for our investigations is the class $\mathcal{UC}$ of *unit-refutation complete clause-sets*, introduced in del Val (1994) as a target class for knowledge compilation. Via the theory of "hardness" of clause-sets as developed in Kullmann (1999), Kullmann (Ann Math Artif Intell 40(3–4):303–352, 2004) and Ansótegui et al. (2008) we obtain a natural generalisation $\mathcal{UC}_k$, containing those clause-sets which are "unit-refutation complete of level k", which is the same as having hardness at most k. Utilising the strong connections to (tree-)resolution complexity and (nested) input resolution, we develop basic methods for the determination of hardness (the level k in $\mathcal{UC}_k$). A fundamental insight now is that $\mathcal{SLUR}_k = \mathcal{UC}_k$ holds for all k. We can thus exploit both streams of intuitions and methods for the investigations of these hierarchies. As an application we can easily show that the hierarchies from Čepek et al. (2012) and Balyo et al. (2012) are strongly subsumed by $\mathcal{SLUR}_k$. Finally we consider the problem of "irredundant" clause-sets in $\mathcal{UC}_k$. For 2-CNF we show that strong minimisations are possible in polynomial time, while already for (very special) Horn clause-sets minimisation is NP-complete. We conclude with an extensive discussion of open problems and future directions. We envisage the concepts investigated here to be

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