J Autom Reasoning (2013) 51:79–108
DOI 10.1007/s10817-013-9281-x

Cutting to the Chase

Solving Linear Integer Arithmetic

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Received: 7 February 2013 / Accepted: 14 February 2013 / Published online: 15 March 2013

Abstract We describe a new algorithm for solving linear integer programming problems. The algorithm performs a DPLL style search for a feasible assignment, while using a novel cut procedure to guide the search away from the conflicting states.

Keywords Linear arithmetic · SMT · SAT · DPLL · Linear programming · Integer arithmetic

1 Introduction

One of the most impressive success stories of computer science in industrial applications was the advent of linear programming algorithms. Linear programming (LP) became feasible with the introduction of Dantzig's simplex algorithm. Although the original simplex algorithm targets problems over the rational numbers, in 1958 Gomory [16] introduced an elegant extension to the integer case (ILP). He noticed that, whenever the simplex algorithm encounters a non-integer solution, one can eliminate this solution by deriving a plane, that is implied by the original problem, but does not satisfy the current assignment. Incrementally adding these *cutting planes*, until an integer solution is found, yields an algorithm for solving linear systems over the integers. Cutting planes were immediately identified as a powerful general tool and have since been studied thoroughly both as an abstract proof system [7], and as a practical preprocessing step for hard structured problems. For such problems,

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