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# E-unification with Constants vs. General E-unification

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**Abstract** We present a solution to Problem #66 from the RTA open problem list. The question is whether there exists an equational theory $E$ such that $E$-unification with constants is decidable but general $E$-unification is undecidable. The answer is positive and we show such a theory. The problem has several equivalent formulations, therefore the solution has many consequences. Our result also shows, that there exist two theories $E_1$ and $E_2$ over disjoint signatures, such that $E_1$-unification with constants and $E_2$-unification with constants are decidable, but $(E_1 \cup E_2)$-unification with constants is undecidable.

**Keywords** E-unification with constants · General E-unification · Combination problem

## 1 Introduction

The aim of the combination problem for unification is to find a procedure which using an $E_1$-unification algorithm and an $E_2$-unification algorithm constructs an $(E_1 \cup E_2)$-unification algorithm. The combination problem was intensively studied by many researchers. The main question is which theories admit a combination procedure. Many results were published for particular types of theories (simple, regular and collapse free, etc.). These results are summarized in [6]. Schmidt-Schauss presented a more general result in [12]. He has not restricted theories to have any particular type, instead he showed that all equational theories $E_1$, $E_2$ over disjoint signatures that have decidable constant elimination problems admit a combination procedure. This result was improved by Baader and Schulz in [3, 4]. They showed that all equational theories $E_1$, $E_2$ over disjoint signatures having decidable $E_1$- and $E_2$-unification

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