## Thermoelectric Module Design Under a Given Thermal Input: Theory and Example

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Established thermoelectric theory enables direct calculation of the power output and conversion efficiency if the temperature difference across a module is given. However, in some applications such as those using a radioisotope or solar radiation as a heat source, the thermal input remains constant while the temperature difference varies with the geometry of the thermoelectric module. In this paper, a theoretical framework for thermoelectric module design under a given thermal input is presented. It provides a convenient approach for module geometry optimization. The usefulness of the theory is demonstrated through a design study, in which an appropriate thermoelement length for a solar thermoelectric system is determined by considering conflicting requirements for a longer length to obtain a greater temperature difference and for a shorter length to produce a larger power output.

**Key words:** Thermoelectrics, thermoelectric generator, thermoelectric energy conversion

## INTRODUCTION

A thermoelectric generator is a solid-state energy convertor that can convert heat into electricity. The theory for thermoelectric devices is well established,<sup>1-4</sup> providing a useful tool for design and evaluation of thermoelectric generators. Based on a simplified model which neglects the influence of thermal and electrical contact resistances, the power output, P, and conversion efficiency,  $\phi$ , of a thermoelectric generator can be determined as<sup>1</sup>

$$P = \frac{s}{\left(1+s\right)^2} \frac{\alpha^2 \Delta T^2}{R_{\rm i}},\tag{1}$$

$$\phi = rac{\Delta T}{T_{
m H}} \left( rac{s}{\left(1+s
ight) - \Delta T/2T_{
m H} + \left(1+s
ight)^2/ZT_{
m H}} 
ight), \quad (2)$$

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where  $s \ (= R_{\rm L}/R_{\rm i})$  is the ratio of the load resistance,  $R_{\rm L}$ , to the internal resistance of the thermoelectric generator,  $R_{\rm i}$ ;  $\alpha$  is the Seebeck coefficient;  $\Delta T \ (= T_{\rm H} - T_{\rm C})$  is the temperature difference across the thermoelectric generator; and Z is the figure of merit of the thermoelectric material. For a given thermoelectric material, Eqs. 1 and 2 enable direct calculation of the performance of a thermoelectric generator if the temperature difference across the generator is known. In general, the temperature difference across a thermoelectric generator can be measured experimentally or assumed to be a variable in the design process. As a result, these equations have been widely employed in the study of thermoelectric generators.

An improved model is also available, which takes into account the effects of thermal and electrical contacts. It provides a more accurate estimate of the power output and conversion efficiency of a thermoelectric module in relation to its geometry. In this more realistic model, the power output and conversion efficiency of a thermoelectric generator can be expressed as<sup>5</sup>

$$P = \frac{s}{(1+s)^2} \frac{\alpha^2}{\rho} \frac{A\Delta T^2}{(n+l)(1+2rl_c/l)^2},$$
 (3)