



# Exponential approximation of the Heidler function for the reproduction of lightning current waveshapes

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## ARTICLE INFO

### Article history:

Received 4 August 2009

Received in revised form 21 April 2010

Accepted 29 April 2010

Available online 26 May 2010

### Keywords:

Exponential approximation

Heidler function

Frequency domain

Time domain

Fourier transform

Lightning protection

## ABSTRACT

In this paper the lightning current function proposed by Heidler is approximated by a linear combination of exponential functions in the time domain, which is afterward analytically transformed into the frequency domain. The unknown coefficients of the exponential approximation for the most frequently used values of the current steepness factor are computed using the least squares method. The developed exponential approximation is general with respect to the current steepness factor in the Heidler function. Approximation of the Heidler function with a linear combination of exponential functions enables the approximation of various Heidler function based lightning current functions. Heidler function in the frequency domain can be used for transient analysis of electromagnetic phenomena that include lightning protection, grounding grid analysis and electromagnetic compatibility problems.

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## 1. Introduction

The lightning stroke models discussed in this paper represent the so-called engineering models which are defined as equations relating the longitudinal current along the lightning channel to the current at the lightning origin, which is usually situated at ground level [1–3]. Analytical expressions are derived from a large amount of lightning parameter data collected through measurements by Berger in Switzerland [4], later on digitized and re-analyzed by Anderson and Eriksson [5,6]. Overview of recent direct current measurements conducted in different parts of world is concisely given in [1].

In the International Electrotechnical Commission (IEC) lightning protection standard [7], a special case of the Heidler lightning current function (HF), for the current steepness factor  $n = 10$ , is used to represent the return-stroke lightning channel-base current. HF with an adjustable current steepness factor [8,9] provides a number of improvements over previously used channel-base lightning current approximation functions such as double-exponential function [10]. The most important improvement is that HF first time derivation for  $t = 0$  is equal to zero and, consequently, does not have a discontinuity at the start. New analytical expressions for lightning current, which are an extension of HF, are introduced by other authors [11–15]. In paper [11], Nucci et al. represent the current at

the channel base of the subsequent return stroke by a combination of the HF with the current steepness factor  $n = 2$  and the double-exponential function. Furthermore, Diendorfer and Uman [12] use a sum of two HFs with the current steepness factor  $n = 2$ . Similar method with the same current steepness factor was proposed by Delfino et al. to represent the current of the first and subsequent return strokes [13]. In paper [14], Cooray et al. use a sum of HF with a large value of the current steepness factor (for first return stroke  $n = 100$ ) and a linear combination of four exponential functions. In paper [15], De Conti et al. use a sum of several HFs with an adjustable current steepness factor. HF based lightning current functions, with  $n = 2$ , are successfully applied to lightning electromagnetic field computation [16,17]. Although the HF introduces a number of improvements over the previous functions of channel-base current and is widely used, its main drawback is the absence of an analytical solution in the frequency domain. In other words, according to the known literature [9], the expression which depicts the HF cannot be integrated analytically via the Fourier transform. Naturally, this becomes a nuisance, when frequency domain computations are required which include the HF.

It is important to mention the lightning current analytical expressions which are a linear combination of exponential functions and have a frequency domain analytical solution: the well-known double-exponential function [10], CBC function [18,19] introduced by Javor and Rancic and NCBC function [20] introduced by Javor as well as so-called pulse function introduced by Feizhou and Shanghe [21]. Besides these, there is also the current waveform recommended by the CIGRE study group [22].

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