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ORIGINAL ARTICLE

The relativistic precession of the orbits

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Abstract The relativistic precession can be quickly inferred from the nonlinear polar orbit equation without actually solving it.

Keywords Polar equation · Angular period operator · Relativistic precession

1 Introduction

The precession, and consequently the secular motion of the pericenter of a bound orbit, stems from the general relativistic treatment of the motion of a test body in the space-time of a spherically symmetric distribution of mass which, if also spins, drags space-time around with it slightly perturbing the orbit (Lense-Thirring gravitomagnetic force), and then by providing a further contribution to the precession. The study of the former phenomenon has been done for various astronomical scenarios. Within the solar system, have been considered Earth's satellites (Rubincam 1977; Cugusi and Proverbio 1978; Ciufolini and Matzner 1992; Iorio et al. 2002, 2004, 2011; Iorio 2008a, 2011a; Lucchesi and Peron 2010; Ashby and Bertotti 1984; Bertotti 1978; Lämmerzahl et al. 2004) the Moon (Iorio 2002; Flechtner et al. 2008), Mars (Iorio 2006) the giant planets (Hiscock and Lindblom 1979; Helled et al. 2011; Helled 2011; Iorio 2007a, 2010) the Sun and its planets (Einstein 1915; Shapiro and et al. 1972; Shapiro et al. 1976; Shapiro 1989; Iorio 2005a, 2005b, 2011b). The 1PN post-Newtonian, Schwarzschild-like orbital effects (Damour and Deruelle 1985) recently revamped because of several attempts to detect them in different natural systems as the galactic environment for stellar orbits around the central black holes (Iorio 2011c; Kannan and Saha 2009; Zakharov et al. 2007). Still in a planetary setting, the exoplanets may constitute a fruitful field of study (Pál and Kocsis 2008; Jordán and Bakos 2008; Iorio 2006, 2011d; Adams and Laughlin 2006a, 2006b, 2006c; Escudé 2002).

The geodetic equation in the Schwarzschild space-time becomes a Binet-type differential equation which describes in polar form the shape of the orbit of a test body under the effect of a force inversely proportional to the square of the distance from the origin with added a very small inverse fourth power term. There is no difficulty in principle, using techniques of perturbation theory, to deduce the precession by solving the equation for bound orbits to arbitrary degrees of approximation (D'Eliseo 2011; Damour and Esposito-Farese 1996; Saca 1995, 2008). Our aim here is to show a new way to obtain the leading term of the precession in two moves, consisting of a substitution followed by an elementary definite integration but, to be able to appreciate its correctness, it needs to be opportunely introduced and justified.

2 The angular period operator

We start from the classical equation of the unperturbed orbit in polar coordinates (D'Eliseo 2007)

$$u'' + u = A, \tag{1}$$

where the primes denote double differentiation with respect to the angle ϕ. The function $u = u(\phi)$ is the inverse of the

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