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ORIGINAL ARTICLE

On the evolution of the '*f*' family in the restricted three-body problem

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Abstract Poincaré surface of section technique is used to study the evolution of a family '*f*' of simply symmetric retrograde periodic orbits around the smaller primary in the framework of restricted three-body problem for a number of systems, actual and hypothetical, with mass ratio varying from 10^{-7} to 0.015. It is found that as the mass ratio decreases the region of phase space containing the two separatrices shrinks in size and moves closer to the smaller primary. Also the corresponding value of Jacobi constant tends towards 3.

Keywords Restricted three-body problem · Poincaré surface of section · Mass ratio · Jacobi constant

1 Introduction

The planar, circular, Restricted Three-Body Problem (RTBP) is a commonly adopted model to study the dynamical evolution of small bodies in the solar system and one of the simplest non-integrable dynamical systems (Szebehely 1967). In order to understand the structure of the solutions, determination of periodic solutions, their stability properties and their structures on the initial conditions space play an important role. Several methods for computing periodic orbits are Lindstedt-Poincaré technique (Szebehely 1967; Celletti 2010 and Viswanath 2001), the Krylov-Bogoliubov-Mitropolsky (KBM) method (Szebehely 1967; Celletti 2010), application of fixed point theorems (Szebehely 1967), method of power series (Szebehely 1967), generating families (Hénon 1997), grid search (Markellos et al. 1974), and many more given by Broucke (1968), Deprit and Rom (1968), Perdios et al. (2002), Gilbout and Scheeres (2003), Kalantonis et al. (2003), Williams (2006) and Tsirogiannis et al. (2009).

Periodic solutions and their stability can also be determined by numerical integration of the equations of motion of the autonomous dynamical system and considering the intersections of the solutions with a surface in the phase-space. This is known as the Poincaré surface of section (PSS) technique (Hénon 1965; Jefferys 1971; Murison 1989; Smith 1991; Tuckness 1995). The set of stable periodic and quasi-periodic trajectories define regions of regular motion or stability "islands" (KAM tori) that spread in a chaotic "sea" made of trajectories with high sensitivity with respect to the initial conditions. The maximum amplitude of oscillation, which is the largest of the quasi-periodic orbits, can be taken as a parameter to measure the degree of stability of the periodic orbit with respect to the region around it in the phase space (Winter 2000). This technique is an easy and powerful tool as it gives a qualitative picture of the stability regions in the planar problem. This technique was applied for finding the location of a family '*f*' of simply symmetric retrograde periodic orbits around the Moon and also their stability in terms of the maximum amplitude of oscillation for Earth-Moon system (mass ratio 0.01215). It was found that at $C = 2.85$

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