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ORIGINAL ARTICLE

Third-order secular Solution of the variational equations of motion of a satellite in orbit around a non-spherical planet

M.K. Ammar · S.M. El Shaboury · M.R. Amin

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Abstract We constructed an analytical theory of satellite motion up to the third order relative to the oblateness parameter of the Earth (J_2). Equations of secular variations was developed for the first three orbital elements (a, e, i) of an artificial satellite. The secular variations are solved in a closed form.

Keywords Satellite earth orbit · Orbit Perturbation · Lagrange equations of motion · Earth oblateness · Secular variations

1 Introduction

There are many analytical theories of a satellite motion implement the calculation of the perturbations due to non-spherical central planet. The most known and precise theories have been worked by Brouwer (1959), Kozai (1962), Kaula (1966), Deprit and Rom (1970), Gaposchkin (1973), Berger (1975), Kinoshita (1977), Chazov (1988), Emelyanove (1991), Blitzer (1960) studied secular and periodic Motions of the node of an artificial Earth-Satellite. King-Hele and Cook (1965), studied the effect of the even zonal harmonics of the Earth's Gravitational Potential on the orbits of Earth satellites.

Few authors worked on the third order perturbations on the motion of artificial satellites, among them; Nasonova (1971), derived an analytical theory based on an intermediate satellite orbit with allowance of second and third harmonics of the geopotential, he obtained the secular perturbations in the angular elements to the third order with respect to the flattening of the planet. Tamarov (1985) obtained the equations of secular perturbations of third order due to the combined effects of a satellite harmonics of the geopotential with numbers $n > 2$.

The objective of this paper concerns with the problem of constructing an analytical theory of satellite motion of third order relative to the oblateness parameter of the Earth.

We derived a secular solution in the first three orbital elements (a, e, i) of an artificial satellite, whose motion is perturbed by the Earth oblateness up to the third order with respect to the oblateness parameter (J_2).

Third-order secular perturbations were derived by using the Taylor expansion for the Lagrange's equations of motion. All quantities were expanded into power series of the eccentricity, but the solution was obtained so as to be closed with respect to the inclination.

2 Formulation of the problem

We will assume that the Earth has rotational symmetry and that the motion of a satellite is governed solely by the Earth's potential function; that is, other forces like drag and attraction of the Sun and Moon are not considered. We introduce a right-handed inertial rectangular coordinate system with the origin at the center of mass of the Earth and the z-axis coincident with the rotational axis and is positive in the north direction. The positive x-axis is chosen in the direction of the

M.K. Ammar (✉)
Math. Dept. Fac. of Sci., Helwan University, P.O. 11957, Cairo, Egypt
e-mail: medhatammar@hotmail.com

S.M. El Shaboury
Math. Dept. Fac. of Sci., Ain Shams University, Cairo, Egypt

M.R. Amin
Theoretical Physics Dept., National Research Center, Cairo, Egypt