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ORIGINAL ARTICLE

Equilibrium points and stability in the restricted three-body problem with oblateness and variable masses

Jagadish Singh · Oni Leke

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Abstract The existence and stability of a test particle around the equilibrium points in the restricted three-body problem is generalized to include the effect of variations in oblateness of the first primary, small perturbations ϵ and ϵ' given in the Coriolis and centrifugal forces α and β respectively, and radiation pressure of the second primary; in the case when the primaries vary their masses with time in accordance with the combined Meshcherskii law. For the autonomized system, we use a numerical evidence to compute the positions of the collinear points $L_{2\kappa}$, which exist for $0 < \kappa < \infty$, where κ is a constant of a particular integral of the Gylden-Meshcherskii problem; oblateness of the first primary; radiation pressure of the second primary; the mass parameter ν and small perturbation in the centrifugal force. Real out of plane equilibrium points exist only for $\kappa > 1$, provided the abscissae $\xi < \frac{\nu(\kappa-1)}{\beta}$. In the case of the triangular points, it is seen that these points exist for $\epsilon' < \kappa < \infty$ and are affected by the oblateness term, radiation pressure and the mass parameter. The linear stability of these equilibrium points is examined. It is seen that the collinear points $L_{2\kappa}$ are stable for very small κ and the involved parameters, while the out of plane equilibrium points are unstable. The conditional stability of the triangular points depends on all the system parameters. Further, it is seen in the case of the triangular points, that the stabilizing or destabilizing behavior of the oblateness coefficient is controlled by κ, while those of the small perturbations depends on κ and whether these perturbations are positive or negative. However, the destabilizing behavior of the radiation pressure remains unaltered but grows weak or strong with increase or decrease in κ. This study reveals that oblateness coefficient can exhibit a stabilizing tendency in a certain range of κ, as against the findings of the RTBP with constant masses. Interestingly, in the region of stable motion, these parameters are void for $\kappa = \frac{4}{3}$. The decrease, increase or non existence in the region of stability of the triangular points depends on κ, oblateness of the first primary, small perturbations and the radiation pressure of the second body, as it is seen that the increasing region of stability becomes decreasing, while the decreasing region becomes increasing due to the inclusion of oblateness of the first primary.

Keywords Celestial mechanics · Gylden-Meshcherskii problem · Oblateness

J. Singh · O. Leke (✉)
Department of Mathematics, Faculty of Science, Ahmadu Bello University, Zaria, Nigeria
e-mail: lekkyonix4ree@yahoo.com

J. Singh
e-mail: jgds2004@yahoo.com

1 Introduction

Mass variable systems have been significant since the foundation of classical mechanics, and have been relevant in modern physics (Lopez et al. 2004). Among these types of systems, we refer to the motion of rockets (Sommerfeld 1964) and black holes formation (Helhl et al. 1998). A satellite moving around a radiating star surrounded by a cloud varies its mass due to particles of this cloud. Comets loose part of their mass as a result of roaming around the Sun (or other stars) due to their interaction with the solar wind which blows off particles from their surfaces (Nuth et al. 2000). The Gylden-Meshcherskii problem, for short GMP (Gylden 1884; Meshcherskii 1902) gives a better insight of a double-star evolution at the secular mass loss owing to photon and corpuscular activity. It is also used as a mathematical model for different cases of a variable mass body