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ORIGINAL ARTICLE

Periodic orbits based on geometric structure of center manifold around Lagrange points

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Abstract This study proposes an analytical method that determines the center manifold and identifies the reduced system on the center manifold. The proposed method expresses the center manifold through general equations containing only state variables, and not functions with respect to time. This is the so-called geometric structure of the center manifold. The location of periodic or quasi-periodic orbits is identified after the geometric structure of the center manifold is determined. The reduced system on the center manifold is described using ordinary differential equations, so that periodic or quasi-periodic orbits can be computed by numerically integrating the reduced system. The results indicate that the analytical method proposed in this study has higher precision compared with the Lindstedt-Poincaré method of the same order.

Keywords Circular three-body problem · Center manifold · Reduced system · Periodic orbit · Quasi-periodic orbit · Analytical method

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1 Introduction

Lagrange point orbits of the restricted three-body problem have received considerable attention in the past few decades. Many scientific missions such as ISEE-3 (1978), WIND (1994), SOHO (1995), ACE (1997), and Genesis (2001) have taken advantage of the privileged location of the L_1 point in the Sun-Earth system (Dunham and Farquhar 2003). The highly stable thermal environment of the L_2 point in the Sun-Earth system has also been selected as the nominal place of many missions, including MAP (2001) (Folta and Richon 1998) and Herschel and Planck (2009) (Juillet et al. 2006).

Such Lagrange point trajectories are generally unstable. Recently, the dynamic system theory has been applied successfully to trajectory designs in the restricted three-body regime (Barden and Howell 1998). Current applications focus on stable, unstable, and center manifolds associated with Lagrange points. The trajectories on the center manifold are bounded, while those on the stable manifold approach exponentially bounded motions and those on the unstable manifold diverge exponentially. Therefore, the motion on the center manifold is a primary concern.

The center manifold has been computed using numerical and analytical methods in previous studies. Quasi-periodic orbits of the center manifold can be determined using a numerical method that employs multiple Poincaré sections (Kolemen et al. 2007). The periodic orbits and invariant 2D tori of the center manifold can be determined using purely numerical procedures (Gómez and Mondelo 2001). Analytical methods are divided into two main procedures: the Lindstedt-Poincaré procedure and the Normal form procedure. The Lindstedt-Poincaré procedure is used to compute invariant manifolds explicitly. Approximations provide valuable insight on the general nature of periodic or