

# Dynamics of modified Chaplygin gas in brane world scenario: phase plane analysis

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**Abstract** In this work we investigate the background dynamics when dark energy is coupled to dark matter with a suitable interaction in the universe described by brane cosmology. Here DGP and the RSII brane models have been considered separately. Dark energy in the form of modified Chaplygin gas is considered. A suitable interaction between dark energy and dark matter is considered in order to at least alleviate (if not solve) the cosmic coincidence problem. The dynamical system of equations is solved numerically and a stable scaling solution is obtained. A significant attempt towards the solution of the cosmic coincidence problem is taken. The statefinder parameters are also calculated to classify the dark energy models. Graphs and phase diagrams are drawn to study the variations of these parameters. It is also seen that the background dynamics of modified Chaplygin gas is completely consistent with the notion of an accelerated expansion in the late universe. Finally, it has been shown that the universe in both the models follows the power law form of expansion around the critical point, which is consistent with the known results.

**Keywords** Dynamical systems

## 1 Introduction

The idea of the static universe once conceived by Albert Einstein faced a big threat when at the turn of the last century observations of Ia Supernova confirmed that our universe is suffering from an accelerated expansion (Perlmutter et al. 1999; Spergel et al. 2003). Now belief and observations are always to be supported by some model. In the quest of finding a suitable model for universe, Cosmologists started to investigate the root cause that is triggering this expansion. Fundamentally, we were to modify Einstein's equation either by modifying the left hand side, i.e., modifying the idea of Einstein gravity or to modify the right hand side which immediately speculate the nature of the matter inside the universe. If our Universe is filled up by some invisible fluid causing a negative pressure then it violates the strong energy condition i.e.  $\rho + 3p < 0$ . Because of its invisible nature this energy component was aptly termed as dark energy (DE) (Riess et al. 2004).

With the introduction of DE, the search began for different candidates that can effectively play the role of DE. DE represented by a scalar field<sup>1</sup> is often called quintessence. Not only scalar field but also there are other Dark fluid models like Chaplygin gas which plays the role of DE very efficiently. The earliest form of this was known as pure Chaplygin gas (Kamenshchik et al. 2001; Gorini et al. 2004). Extensive research saw pure Chaplygin gas first modify into generalized Chaplygin gas (Gorini et al. 2003; Alam et al. 2003; Bento et al. 2002; Carturan and Finelli 2003;

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<sup>1</sup>In the presence of a scalar field the transition from a universe filled with matter to an exponentially expanding universe is justified.