

# (An)Isotropic models in scalar and scalar-tensor cosmologies

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**Abstract** We study how the constants  $G$  and  $\Lambda$  may vary in different theoretical models (general relativity with a perfect fluid, scalar cosmological models (“quintessence”) with and without interacting scalar and matter fields and a scalar-tensor model with a dynamical  $\Lambda$ ) in order to explain some observational results. We apply the program outlined in section II to study three different geometries which generalize the FRW ones, which are Bianchi V, VII<sub>0</sub> and IX, under the self-similarity hypothesis. We put special emphasis on calculating exact power-law solutions which allow us to compare the different models. In all the studied cases we arrive at the conclusion that the solutions are isotropic and noninflationary while the cosmological constant behaves as a positive decreasing time function (in agreement with the current observations) and the gravitational constant behaves as a growing time function.

**Keywords** Scalar and Scalar-tensor cosmologies · Self-similar solutions · Bianchi models

## 1 Introduction

In a series of recent papers (Belinchón 2011a, 2011b) we have studied and compared how the “constants”  $G$  and  $\Lambda$  may vary in different theoretical frameworks for several metrics. These different theoretical frameworks are: general relativity with time varying constants (TCV), scalar cosmological models with non-interacting scalar and matter fields and TVC, and the last of the studied models is the usual

scalar-tensor theory with a dynamical cosmological constant which seems to be the most natural theoretical model to study the possible variation of the gravitational and the cosmological constants. In those recent works we have been able to state and prove general results valid for all the geometries (all the Bianchi types as well as for the FRW geometries) within the context of self-similar solutions (SSS).

We have focused our attention on this class of solutions since as has been pointed out by Rosquist and Jantzen (1985) self-similar models, correspond to equilibrium points, playing a dominant role in the dynamics of Bianchi cosmological models. For this reason, Coley (2003), has stressed the fact that self-similar models play an important role in describing the asymptotic dynamics of the Bianchi models. A large class of orthogonal spatially homogeneous models (including all class B models) are asymptotically self-similar at the initial singularity and are approximated by exact perfect fluid or vacuum self-similar power-law models. In the same way, exact self-similar power-law models can also approximate general Bianchi models at intermediate stages of their evolution and which is also important, self-similar solutions can describe the behaviour of Bianchi models at late times. Furthermore working under the hypothesis of self-similarity allows us to find exact power-law solutions in such a way that we may compare the obtained solution for each studied model.

In this paper we extend our program by considering the important case of the interacting scalar and matter fields within the framework of scalar cosmological models, and of course, we formulate the corresponding model with TVC. Therefore the aim of the present work consists in studying and comparing, by calculating the exact solution, how the constants  $G$  and  $\Lambda$  may vary in different theoretical models and with different geometries under the self-similar hypothesis. We apply the outlined program to study three dif-

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