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ORIGINAL ARTICLE

Regularization of the circular restricted three-body problem using 'similar' coordinate systems

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Abstract The regularization of a new problem, namely the three-body problem, using 'similar' coordinate system is proposed. For this purpose we use the relation of 'similarity', which has been introduced as an equivalence relation in a previous paper (see Roman in Astrophys. Space Sci. doi:10.1007/s10509-011-0747-1, 2011). First we write the Hamiltonian function, the equations of motion in canonical form, and then using a generating function, we obtain the transformed equations of motion. After the coordinates transformations, we introduce the fictitious time, to regularize the equations of motion. Explicit formulas are given for the regularization in the coordinate systems centered in the more massive and the less massive star of the binary system. The 'similar' polar angle's definition is introduced, in order to analyze the regularization's geometrical transformation. The effect of Levi-Civita's transformation is described in a geometrical manner. Using the resulted regularized equations, we analyze and compare these canonical equations numerically, for the Earth-Moon binary system.

Keywords Restricted problems: restricted problem of three bodies · Stellar systems: binary stars · Methods: regularization

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1 Introduction

In a previous article (see Roman 2011), by introducing the "similarity" relation and applying it to the restricted three-body problem, the "similar" equations of motion were obtained. These equations were connected with the classical equations of motion by some coordinate transformation relations (see equations (17) in Roman 2011). In this paper were also defined 'similar' parameters and physical quantities, and 'similar' initial conditions and some trajectories of the test particles into the physical (x, S_1, y) and respectively (x, S_2, y) planes were plotted.

Denoting S_1 and S_2 the components of the binary system (whose masses are m_1 and m_2), the equations of motion of the test particle (in the frame of the restricted three-body problem) in the coordinate system (x, S_1, y, z) are (see equations (11)–(13) in Roman 2011):

$$\frac{d^2x}{dt^2} - 2\frac{dy}{dt} = x - \frac{q}{1+q} - \frac{x}{(1+q)r_1^3} - \frac{q(x-1)}{(1+q)r_2^3}, \qquad (1)$$

$$\frac{d^2y}{dt^2} + 2\frac{dx}{dt} = y - \frac{y}{(1+q)r_1^3} - \frac{q\,y}{(1+q)r_2^3}, \qquad (2)$$

$$\frac{d^2z}{dt^2} = -\frac{z}{(1+q)r_1^3} - \frac{q\,z}{(1+q)r_2^3}, \qquad (3)$$

where

$$r_1 = \sqrt{x^2+y^2+z^2}, \qquad (4)$$
$$r_2 = \sqrt{(x-1)^2+y^2+z^2}, \quad q = \frac{m_2}{m_1}.$$

In the 'similar' coordinate system (x', S_2, y', z') the equations of motion of the test particle are (see equations